Signal processing and feature extraction by using real order derivatives and generalised norms. Part 2: Applications

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A combined signal processing and feature extraction approach can be based on generalised moments $\mathcal{M}_a^\alpha$ and norms, $\| M_a^\alpha \|_p = \left( \mathcal{M}_a^\alpha \right)^{1/p} = \left( \frac{1}{N} \sum_{n=0}^{N-1} x(n)^{(p\alpha)} \right)^{1/p}$, which are defined by the order of derivation ($\alpha$), the order of the moment ($p$) and sample time ($\tau$), where $a$ and $p$ are real numbers. The analysis can be further improved by taking into account non-linear effects by monotonically increasing scaling functions. Several features can be combined in condition indices and, in some cases, only one feature is needed if the orders $p$ and $a$ are chosen properly. The aim of the application analysis is to test new features and whether their sensitivity is sufficient to detect faults from the absolute values of the dynamic part of the signals $x^{(\alpha)}$ at an early stage when the faults are still very small. For faults causing impacts, the sensitivity of the features is clearly improved by higher and real order derivation, and low order norms can be used if the order of derivation is sufficient. Correspondingly, subharmonic vibrations can be amplified by integration. A limit of sensitivity is reached on a certain order $a$, and on order $a$ an optimal sensitivity is chosen by the order $p$. Both the orders can be chosen fairly flexibly from the optimal area. Sample time, which connects the features to the control application, is process-specific. The analysis methods allow the use of lower frequency ranges, and a multisensor approach can also be used. The approach is well suited for rotating process equipment with speeds ranging from very slow to very fast. In this article, the types of slow rotating equipment are a lime kiln, a washer and the scraper of a digester, all from pulp mills. A centrifuge and a turbo compressor are examples of very fast rotating machines. Features can also be combined in stress indices, which react to harmful process conditions. For example, strong cavitation, cavitation-free cases and even short-term cavitation are clearly detected in a Kaplan water turbine. Several features, also from different frequency ranges, can be combined in condition and stress indices.

**Keywords:** Higher and real order derivatives, vibration analysis, feature extraction, $l_p$ norms, condition indices, cavitation, lime kilns, rolling bearings and condition monitoring.

1. **Introduction**

The mathematical background of fractional derivatives and feature extraction methods are discussed in (10). Unbalance and misalignment can be detected successfully with displacement $x(t)$ and velocity $x^{(1)}$. They do not usually allow the detection of impact-like faults, for example defective rolling bearings and gears, at a sufficiently early stage. The properties of displacement and velocity transducers are discussed in (22). The signals $x$ and $x^{(1)}$ can also be obtained from the acceleration $x^{(2)}$ through analogue or numerical integration ($14,15,16$). High-frequency vibrations are related to bearing faults ($5,6$). Higher order derivatives introduce additional methods for vibration analysis ($23,24$). The first time derivative of acceleration, the jerk, has been used for assessing the comfort of travelling, for example in designing lifts, and for slowly rotating rolling bearings ($9$). In slowly rotating machines, the acceleration pulses are usually weak and occur at long intervals, and the changes in acceleration are rapid and become emphasised upon derivation of the signal $x^{(2)}$. The integration of displacement introduced in ($17$) and complex order derivatives introduced in ($18$) offer additional possibilities for signal processing.

In vibration analysis, root mean square (rms) and peak values are the most commonly used features ($12,13$). Dimensionless features are obtained by normalisation. The normalised moment corresponding to order four, known as kurtosis, is a widely used special case. The kurtosis of acceleration provides an early warning of bearing faults, for example, since it often has a higher sensitivity than the corresponding rms and peak values ($12,13$). Higher order moments were utilised for bearing faults in ($14,15$) and for gears in ($19$). Spectral kurtosis, which is based on kurtosis values in different frequencies, has been used for rotating machines ($17,18,19$). In a centrifuge application ($20$), the condition assessment was based on the rms value of $x^{(2)}$ and the spectra of $x^{(1)}$ and $x^{(2)}$. In water turbines, statistical distributions were already used in the analysis of acceleration signals ($21$) in the 1970s. Cavitation and the avoidance of cavitation in water turbines are reported in ($22,23,24$). Detecting faults in the supporting rolls of lime kilns is usually based on acoustic emission and acceleration signals, see ($25$).

In time domain analysis, special methodologies have been developed to provide efficient online measurements for the peak value, which is a good feature for impact-like faults.
shock pulse method (SPM) patented in 1966 uses signal amplification around the resonance frequency of the transducer (~32 kHz)\(^{(28)}\). In the PeakVue method the time domain signal is created using the peak values obtained from each short sample\(^{(27)}\). Time domain envelope signals are formed using rectification\(^{(28)}\). Spike energy, which is a measure of the intensity of energy generated by repetitive transient mechanical impacts, is used in the ultrasonic region\(^{(29)}\). Wavelet analysis provides features related to local basis functions in the time domain\(^{(30,31)}\). Wavelet-based Holder regularity analysis has been used in fault detection for roller bearings\(^{(32)}\). Time synchronous averaging is used to extract that part of a signal which has the same period as a trigger signal\(^{(33)}\).

The moments and norms can be generalised to real-valued orders if all the signal values are positive. The generalised central moment introduced in\(^{(34)}\) works well, even with short sample times. The generalised norms introduced in\(^{(35)}\) have the same dimensions as the signal to be analysed\(^{(1)}\). The vibration indices introduced in\(^{(17)}\) combine several higher derivatives in different frequency ranges. Dimensionless vibration indices can be combined in measurement indices\(^{(36)}\). Intelligent condition indices are based on the non-linear scaling that was developed to extract the meanings of variables from measurement signals\(^{(38,37)}\). The non-linear scaling approach was first used for the data obtained from a test-rig of roller bearings\(^{(31)}\). The combined approach was summarised in\(^{(48)}\).

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Feature extraction is based on velocity \(x^{(1)}\), acceleration \(x^{(2)}\), higher derivatives \(x^{(3)}\) and \(x^{(4)}\), and real order derivatives \(x^{(\alpha)}\), \(\alpha \in \mathbb{R}\). The other signals have been obtained from acceleration through analogue\(^{(50)}\) or numerical integration and derivation\(^{(51,52)}\). The aim of condition monitoring is to detect faults at an early stage, and vibration analysis usually deals with the dynamic part of the signal. In a sufficiently long signal, for example, the mean value of signals \(x^{(1)}\) and \(x^{(2)}\) is zero. Otherwise the measurement point, for example the bearing housing, would move away from the machine. The assumption \(\overline{x^{(0)}} = 0\) means that the root mean square of \(x^{(0)}\) is equal to the standard deviation: \(x^{(0)} = \sigma_{x^{(0)}}\).

\section{Order of derivation}

The calculation of the time domain signal \(x^{(\alpha)}(t)\), which is based on a rigorous mathematical theory\(^{(37)}\), is performed in three steps\(^{(1)}\). The fast Fourier transform (FFT) is first used for the signal \(x(t)\) to obtain the complex components \(X(k)\), \(k = 0, 1, 2, \ldots, (N - 1)\). The corresponding components of the derivative \(x^{(\alpha)}(t)\) are calculated as follows:

\[ X_{nk} = (i\alpha \omega)^n X_k \]  

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Finally, the resulting sequence is transformed with the inverse Fourier transform FFT\(^{-1}\), which produces the signal \(x^{(\alpha)}(t)\).

Derivation and integration can be performed with both an analogue and a digital technique. In the applications\(^{(50,58)}\), acceleration signals have been recorded in the frequency range from 10 Hz to 20 kHz. The linear range of the analogue differentiator/integrator was from 2 to 2000 Hz. The equipment had a low-pass filter with a cut-off frequency of 2000 Hz. Sharp band-pass filtering was applied to the analogue velocity signal whose frequency range was from 10 or 100 Hz to 1000 Hz. After filtering to the linear range, the resulting signals were transferred to a computer by means of a data acquisition card\(^{(50)}\). In the digital approach, the recorded acceleration signals were transferred to a computer and derivated or integrated numerically with LabVIEW, and all the signals were filtered by means of a sixth-order Butterworth band-pass filter\(^{(52)}\). A combined analogue and digital technique can markedly reduce the amount of data. This is important in intelligent sensors, especially if the rotation speed is very low. We can perform an analogue derivation and rectification to obtain the \(x^{(4)}\) envelope spectrum.
2.2 Generalised moments

The generalised central absolute moment about zero can be normalised by means of the standard deviation $\sigma_x$ of the signal $x^{(a)}$:

$$m^p_i = \frac{1}{N(\sigma_x^p)} \sum_{i=1}^{N} |x^{(a)}_i|^p = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{x^{(a)}_i}{\sigma_x} \right]^p \quad \cdots (2)$$

which was presented in (5). The real number $a$ is the order of derivation and the real number $p$ is the order of the moment. The moment is obtained from the absolute values of signals $x^{(a)}$. The signal is measured continuously and the analysis is based on consecutive equally-sized samples. The duration of each sample is called the sample time, denoted by $\tau$. The number of signal values $N = \tau N_s$ where $N$ is the number of signal values which are taken in a second. The peaks of the signal have a strong effect on the moment (2), which can be used in the same way as kurtosis (10).

2.3 Generalised norms

The $l^p$ norm of $x^{(a)}$ is defined by:

$$\|x^{(a)}\| = \|M^p_i\| = \left( \sum_{i=1}^{\tau} [M^p_i] \right)^{1/p} = \left( \frac{1}{N} \sum_{i=1}^{N} |x^{(a)}_i|^p \right)^{1/p} \quad \cdots (3)$$

where $p \geq 1$ has the same dimensions as the corresponding signals $x^{(a)}$. This norm combines two trends: a strong increase caused by the power $p$ and a decrease with the power $1/p$. The norm (3) can be generalised to represent the norms from the minimum to the maximum, which correspond to the orders $p = -\infty$ and $p = \infty$, respectively. The norm (3) includes the absolute mean ($p = 1$) and the rms value ($p = 2$) as special cases. The norms can be obtained as the norm for the norms of individual samples. A feature can also be defined as a maximum of the norms $\{M^p_i\}$ calculated from different samples $i = 1, \ldots, K_s$, i.e.:

$$\max(\|M^p_i\|) = \max \left\{ \{M^p_i\} \right\} \quad \cdots (4)$$

The number of signal values in each sample is equal and defined by the sample time and the number of signal values in a second. The sample time $\tau$ is an essential parameter in the calculation of moments and norms (1). A summary of our long experience about the applicability of features is presented in (11). Some faults, such as unbalance, misalignment, bent shaft and mechanical looseness, can be detected by means of displacement $x^{(3)}$ and velocity $x^{(4)}$. The norms with order 2 and $\infty$ are most commonly used (Table 1). For an electric motor, the feature $x^{(4)}_b = \|M^p_i\|$ provides information on unbalance, misalignment, mechanical looseness and some electrical faults.

On the other hand, faults causing impacts can be detected more efficiently with acceleration or higher order derivatives. Higher order derivatives provide more sensitive solutions, i.e. the ratios of features between the faulty and non-faulty cases become higher, and additional flexibility can be achieved with the real order of derivation (1). For rolling bearing faults, displacement and velocity should be replaced by acceleration. For a very low rotation speed, the rms values are not sensitive for bearing faults because the effect of a few weak impacts is small in the sum (3) when $N$ is a large number and $p = 2$. For high rotation speeds, frequent strong impacts affect rms values significantly. The rms values can then be used to detect rolling bearing faults, especially if $a \geq 2$. The crest factor combines two features as it is the ratio of the peak value and the rms value.

Rolling bearing faults in slowly rotating machinery can be detected with the maximum norm, which in diagnostics is called peak value. An efficient solution is to use peak values $x^{(2)}_b$, $x^{(3)}_b$ and $x^{(4)}_b$. To avoid the domination of a distinct peak, the peak value can be calculated, for example, as an average of the highest three peaks. The feature $x^{(4)}_b = \|M^p_i\|$ reacts to rolling bearing faults, lubrication problems and stator coil faults. The feature $x^{(4)}_b$ can be used for detecting cavitation in a pump as well.

Table 1. Examples of signals and features in fault detection (11)

<table>
<thead>
<tr>
<th>Nature of fault</th>
<th>Signal</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Unbalance</td>
<td>$x^{(1)}$, $x^{(3)}$</td>
<td>rms, peak</td>
</tr>
<tr>
<td>2. Misalignment</td>
<td>$x^{(1)}$, $x^{(3)}$</td>
<td>rms, peak</td>
</tr>
<tr>
<td>3. Bent shaft</td>
<td>$x^{(1)}$</td>
<td>rms, peak</td>
</tr>
<tr>
<td>4. Damaged rolling element bearings</td>
<td>$x^{(2)}$, $x^{(3)}$, $x^{(4)}$</td>
<td>peak, rms, crest factor, kurtosis, $l^p$ norm</td>
</tr>
<tr>
<td>5. Mechanical looseness</td>
<td>$x^{(1)}$</td>
<td>rms, peak</td>
</tr>
<tr>
<td>6. Damaged or worn gears</td>
<td>$x^{(2)}$, $x^{(3)}$, $x^{(4)}$</td>
<td>peak, rms, $l^p$ norm</td>
</tr>
<tr>
<td>7. Oil whirl</td>
<td>$x^{(1)}$, a &lt; 0, $x^{(3)}$</td>
<td>rms, peak</td>
</tr>
<tr>
<td>8. Resonance</td>
<td>$x^{(1)}$</td>
<td>rms, peak</td>
</tr>
<tr>
<td>9. Poor lubrication</td>
<td>$x^{(2)}$, $x^{(3)}$, $x^{(4)}$</td>
<td>peak, rms, $l^p$ norm</td>
</tr>
<tr>
<td>10. Cavitation</td>
<td>$x^{(2)}$, $x^{(3)}$, $x^{(4)}$</td>
<td>peak, rms, $l^p$ norm</td>
</tr>
<tr>
<td>11. Electrical problems</td>
<td>$x^{(1)}$</td>
<td>rms, peak</td>
</tr>
<tr>
<td>12. Loose stator coils</td>
<td>$x^{(2)}$, $x^{(3)}$, $x^{(4)}$</td>
<td>rms, peak</td>
</tr>
</tbody>
</table>

The distributions of the signals $x^{(1)}$, $x^{(3)}$ and $x^{(4)}$ have been used in monitoring the condition of the supporting rolls of a lime kiln (53,58). The bins of the histograms are based on the standard deviation $\sigma_x$ of the corresponding signal $x^{(a)}$.

2.4 Condition and stress indices

Vibration signals can be utilised in process or machine operation by combining features obtained from derivatives. A weighted sum of the norms obtained for different orders of derivatives is a norm as well. Each norm has a weight factor to make the sum dimensionless (10). The inverse of the norm provides an indication of the health of the machine. Dimensionless vibration indices can be combined in a measurement index:

$$\tilde{M} = \frac{1}{n} \sum_{i=1}^{n} b_i \left[ \frac{|x^{(a)}_i|}{|x^{(a)}_b|} \right] \quad \cdots (5)$$

where the norms $\|x^{(a)}_{\alpha} \|$ are obtained using the signals $x^{(a)}_i$, $i = 1, \ldots, n$. Each norm is divided by its reference value, denoted by the index zero, and multiplied by a weight factor $b_{\alpha}$. The sum $\sum b_{\alpha} = n$. The reference values correspond to good conditions.
The inverse of the index MIT, denoted as SOL, provides a direct indication of the condition of the machines: small values indicate poor condition and high values good condition\(^{(4)}\). The machine is in good condition if all the terms of MIT are equal to one. In this case SOL = MIT = 1, and weakening health is seen when the SOL index decreases from one. These indices are not restricted to vibration signals\(^{(4)}\).

The analysis can be further improved by taking into account non-linear effects\(^{(38,50,54,58)}\). The scaling function scales the real values of variables to the range of \([-2, 2]\) with two monotonously increasing functions: one for the values between \(-2\) and 0, and one for the values between 0 and \(2\)\(^{(48)}\). Non-linear scaling has been used for statistical features\(^{(53,58)}\) and features based on the signal distribution\(^{(53,54)}\). A condition index can be based on several features, which are all scaled to the range \([-2, 2]\):

\[
F_{c,k}^{(x)} = \sum_{k=1}^{n} w_k f_k^{-1}\left[\max\left(\left\|M_{x,p}^{(k)}\right\|\right)\right] \cdot \sum_{k=1}^{n} w_k f_k^{-1}(F_{c,k}^{(x)}) \quad \ldots \quad (6)
\]

where \(w_k\) is the coefficient and \(f_k^{-1}\) the scaling function of the feature \(k\). Features include maximum norms \(\max\left(\left\|M_{x,p}^{(k)}\right\|\right)\), and other features \(F_{c,k}^{(x)}\), for example bins of the histograms. Features can have specific frequency ranges. Index (6) is obtained from the features of the signal \(x^{(a)}\), but an index can also combine the features of different physical signals.

3. Experiments with test-rigs

An inner race fault in a spherical double row roller bearing SKF 24124 CC/W33 has been analysed in\(^{(48)}\). For the rotation frequency 2.0 Hz, the maximum sensitivity 7.00 was achieved with peak value (\(p = \infty\)) obtained for \(a = 4.75\) (Figure 1). Almost as high a sensitivity was achieved with the kurtosis: the sensitivity of the kurtosis became three times higher compared with the acceleration (\(a = 2\)). The maximum is achieved when \(a = 4.5\). The severity of faults can be assessed by comparing the features on a different order of derivation (Figure 1). In this case, the measurements were performed in the frequency range 3-2000 Hz. Measurements in a wider rotation frequency range 0.5-12.5 Hz have also been studied.

A multisensor approach has been used for fault diagnosis in a test-rig that consists of an electric motor and a transmission between two axes with SKF single row ball bearings 6002\(^{(38)}\). Two sensors measured axial vibration and five accelerometers radial vibration in the vertical direction. The rig was to simulate three coupling misalignment cases between the motor and input shaft, bent shaft and three bearing faults. One fault at a time was simulated in five rotation frequencies and vibration data were collected by means of seven acceleration sensors. The speed of the secondary shaft is 0.847 times the rotation speed of the motor, which was from 15 to 19 rps. The offset was removed from the signals before calculating the features: rms and kurtosis of the acceleration \(x^{(a)}\), the average of the highest three values of the jerk \(x^{(j)}\), and rms velocities \(x^{(p)}\) in two frequency ranges, 10-1000 Hz and 20-85 Hz. The order \(a\) was 1, 2 and 3. The order \(p\) is 2 in rms, and a moment of order 4 is used in kurtosis. The average of the highest three values corresponds to a very high order \(p\).

The test-rig has been used for testing advanced modelling\(^{(38)}\) and tuning methodologies, including immune systems\(^{(44)}\) and genetic algorithms\(^{(46)}\). Excellent classification results have been achieved in\(^{(38)}\) with the system, where each case model consists of seven equations developed for feature groups of six features. The number of sensors used to diagnose each fault mode were reduced in\(^{(46)}\).

For a roller contact on a rough surface of an inclined plane with a scratch, the maximum sensitivity of the kurtosis, which was obtained with \(a = 3.75\), was as much as 10.9 times higher than for \(a = 2\), see\(^{(46)}\). For peak values, the change is smaller but already starts at \(a = 3\). Reliable results can be obtained by norms \(\left\|M_{x,p}^{(k)}\right\|\) if \(a\) and \(p\) are in the range between 4 and 6.

4. Applications

Methodologies have been developed in the following applications: the scraper of a continuous digester\(^{(48)}\), a washer\(^{(48)}\), the gearbox of a sea water pump\(^{(35)}\), a turbo compressor system\(^{(8,49)}\), a Kaplan water turbine\(^{(34,35,37,50)}\), a lime kiln\(^{(53,58)}\) and a centrifuge\(^{(54)}\).

4.1 Scraper of a continuous digester

The peak value \(x^{(p)}\) measured on the bearing housing of the bottom scraper of a continuous digester in a pulp mill decreased by a factor of 5.36 after the replacement of two faulty bearings during a normal maintenance stoppage\(^{(48)}\). Since the incipient faults were detected at an early stage they did not cause any unscheduled production interruptions. The speed of rotation was 6 r/min corresponding to the frequency 0.1 Hz, which was earlier thought to be the lower limit for detecting roller bearing faults with vibration analysis. The height of the digester is over 50 m. The measurements were performed in the frequency range 5-6500 Hz.

4.2 Oil whirls

Oil whirls are not always easy to detect in displacement or velocity signals, since the increase of the whirl may be hidden under the vibrations caused by the unbalance. These subharmonic vibrations can be amplified by integration. The oil whirl in frequency range 0.42...0.48 times rotation frequency can be detected better if \(a < 0\)\(^{(49)}\). In the sleeve bearing of the turbo compressor system studied in\(^{(49)}\), an oil whirl was identified with \(x^{(a)}\) when the order \(a < -2.5\) (Figure 1). The sensitivity increases with decreasing order and reaches the maximum when \(a = -4.75\). The measurements were performed in an oil refinery in the frequency range 45-1000 Hz. The shaft had a fast rotation frequency of 122 Hz. The rms value \(x^{(a)}\) was calculated from the velocity spectrum.
4.3 Washer

Very slowly rotating bearings in a washer in a pulp mill were analysed in(34). The rotation frequency 0.0254 Hz is well below the frequency 0.1 Hz. The diameter of the inner race of the bearing was about 800 mm. As there are very few peaks, the rms values \(x_{\text{rms}}\) are very insensitive to the faults in this case (Figure 2(a)). The best sensitivity was achieved with the peak values \(x_{\text{peak}}\) (Figure 2(b)). The crest factor \(x_{\text{crest}}/x_{\text{rms}}\) is useful as well (Figure 2(c)). The sensitivity increases with increasing order \(a\) in Figures 2(b) and 2(c). In this case, the peak \(x_{\text{peak}}\) value obtained from the frequency range 10-2000 Hz is already sufficient for early detection.

4.4 Gears

Faults of gears have been studied in an oil refinery. The seawater pump of a cooling system was driven by a 1.1 MW motor and a gearbox with a gear mesh frequency of 422 Hz is presented as an example in(35). Two faulty teeth were observed in the first gear and the results of the inspection measurements, which were performed after the gears had been replaced, are used here as reference values. The measurement index values obtained from rms values \((p = 2)\) increase from 1.16 to 1.83 when the order \(a\) increases from one to three. For the peak value \((p = \infty)\), the corresponding index values are higher, increasing from 1.28 to 2.65. Also, the combined index:

\[
MIT_{\alpha}(a,\alpha) = \left[\frac{1}{2}\left(MIT_{\alpha} + \text{MIT}_{\alpha}\right) + \left(\frac{\alpha}{\sigma}\right)^2 \right] \quad \cdots \quad (7)
\]

has similarly increasing values (Table 2). Weight factors \(b_1 = b_\alpha = 1\) were used in the calculations. The weight factors can be adjusted on the basis of accumulated knowledge. Measurement and health indices are useful, especially if there are two or more simultaneous faults.

4.5 Cavitation

In the cavitation studies, the Kaplan water turbine had sleeve bearings and four blades. The turbine operates with a constant rotation speed, in this case 115 r/min, and the power is controlled by changing the volume flow rate of water. The acceleration signals obtained from the supporting bearings were analysed by LabVIEW with \(N = 12800\) Hz at 29 power levels between 1.5 and 59.4 MW. Both analogue and numerical derivation and integration have been used for signals(50,52). In the numerical derivation and integration of the acceleration signals, the frequency ranges were 10-1000 Hz, 10-2000 Hz, 10-3000 Hz and 10-4000 Hz(53).

The cavitation analysis has been performed with several methods for the signals \(x^{(i)}\), the mean \(x^{(\alpha)}\) was subtracted from the signals. In(50) the analysis was based on two features: the mean peak \(x_{\text{mp}}^{(\alpha)}\) and the fraction \(F_{\alpha}^{(p)}\) of the peaks exceeding the normal range \([-3\sigma_{\alpha}, 3\sigma_{\alpha}]\) obtained from the signal \(x^{(\alpha)}\). \(a = 1, 3\) and 4. The mean peak \(x_{\text{mp}}^{(\alpha)}\) is a mean of the ten highest signal values. The velocity \(x^{(i)}\) was replaced by the acceleration \(x^{(3)}\) in(53,52). The generalised central absolute moment about \(c = x^{(\alpha)}\) was introduced in(54).

The moment \(2\) includes normalisation by the standard deviation \(\sigma\). In addition to the orders \(a\) and \(p\), a new parameter was introduced: the sample time \(\tau\) connects the moment \(2\) to control applications. The \(I_p\) norm defined by (3) was introduced in(55).

The relative \(\max|\sum|M_{\alpha}^{(p)}|\) shown in Figure 3 is the maximum norm (4) divided by the corresponding value of a cavitation-free case at 15 MW. The strong increase caused by the power \(p\) is compensated by a decrease with the power \(1/p\). The strong cavitation points are clearly seen when the order \(p\) increases. The norm in Figure 3 increases with increasing the order \(p\), though much more slowly in the high power range than in the low power range.

The order \(p\) was compared in the range from 0.25 to 8 with a step of 0.25, and a total of 11 sample times were used: \(\tau = 1, 2, \ldots, 6, 8, 10, 20, 30\) and 40 seconds. The length of the signals \(x^{(i)}\) was 50 seconds.

The sensitivities of the special cases of the norm (3) are consistent with the results presented in(50,52): the rms value \((p = 2)\) works well in the high power range and the peak value \((p = \infty)\) in the low power range (Figure 3). As the peak values are based on the highest three peaks in the discretised values, the three values may also originate from a single peak. The kurtosis is a useful feature in the low power range but, for the cavitation-free area and the high power range, the kurtosis is close to value 3, which corresponds to a Gaussian signal, ie kurtosis does not give an indication of cavitation in the high power range. The

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**Table 2. Measurement index MIT\(_{\alpha}(a,\alpha)\) and health index SOL for the gears of a seawater pump**

<table>
<thead>
<tr>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(x_{\text{rms}}^{(m)})</th>
<th>(x_{\text{rms}}^{(i)})</th>
<th>(x_{\text{rms}}^{(\text{peak})})</th>
<th>(x_{\text{rms}}^{(\text{peak})})</th>
<th>MIT(_{\alpha}(a_1,\alpha_2))</th>
<th>SOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.19 mm/s</td>
<td>1.03 mm/s</td>
<td>3.67 mm/s</td>
<td>2.87 mm/s</td>
<td>1.22</td>
<td>0.822</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1.19 mm/s</td>
<td>1.03 mm/s</td>
<td>20.6 m/s(^2)</td>
<td>10.2 m/s(^2)</td>
<td>1.59</td>
<td>0.630</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1.19 mm/s</td>
<td>1.03 mm/s</td>
<td>217 km/s(^3)</td>
<td>81.9 km/s(^3)</td>
<td>1.90</td>
<td>0.526</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5.15 m/s(^2)</td>
<td>3.21 m/s(^2)</td>
<td>20.6 m/s(^2)</td>
<td>10.2 m/s(^2)</td>
<td>1.81</td>
<td>0.552</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>46.6 km/s(^3)</td>
<td>25.5 km/s(^3)</td>
<td>217 km/s(^3)</td>
<td>81.9 km/s(^3)</td>
<td>2.24</td>
<td>0.447</td>
</tr>
</tbody>
</table>
sensitivity of the moment (2) improves when the order $p$ of the moment increases, especially when a short sample time $\tau$ is used. Good results were obtained with moments where $a = 4$, $p = 4$ and $\tau = 3s^{(34)}$. Thus, the maximum of the kurtosis coefficients obtained from ten short samples is a good feature. As the values of these moments are much higher for the cavitation cases in the low power range than in the high power range, the power range needs to be taken into account.

The full power range can be handled by combining two specialised features$^{(34)}$: peak values, kurtosis or normalised moments with high order $p$ for the lower power range, and the high power range is handled with rms values, normalised moments with low order $p$ or the fractions $F_h^{(50)}$ introduced in$^{(50)}$. These fractions have low values in the low power range where the spikes are less frequent. The values rise with increasing power as the number of small spikes grows. This can be heard as an increasing noise.

The cavitation-free conditions and both strong and short term cavitation in the high power range can be detected with the relative max$^{(4)}$ $M_4^p$, the result is not sensitive to the order $p$ (Figure 4(a)). Even the absolute average $\|M_4^p\|$ can be used in this range. The full power range can be analysed by the relative max$^{(4)}$ $M_4^{2.75}$ or by a sum of two relative max$^{(4)}$ $M_4^p$ norms: one obtained for a low and one for a high order $p$.

The results in the low power range are very sensitive to the order $p$ and the sample time $\tau$, see Figure 4. The optimum order was selected by comparing it to the knowledge-based cavitation index$^{(35)}$. The coefficient of determination presented in Figure 5 provides a measure for predicting the knowledge-based cavitation index with the relative max$^{(4)}$ $M_4^p$. The hedge of $R^2$ means that there is an optimum order $p$ for each sample time $\tau$. The order $p$ of the norm was set at 2.75 and sample time $\tau = 3s$, which provides a good balance between low and high power ranges.

Cavitation-free conditions were reliably detected with features obtained in the frequency range 10-1000 Hz. The power range, which is free of cavitation, was taken as a basis for detecting an increase in the signal levels. Also, the jerk $x^{(3)}$ works well in this area. The cavitation cases are more problematic: the norms with $a = 3$ are unacceptable in the low power range and especially in the high power range. For the relative max$^{(4)}$ $M_4^p$, the $R^2$ value is improved in the low power range when a higher value of the order $p$ was used. The most difficult part is to find differences between cavitation and short-term cavitation. Although the acceleration features provided good fits with the training and test data, they are not sensitive enough for detecting operating conditions. The features of the higher derivatives $x^{(3)}$ and $x^{(4)}$ have much better overall performance. The features of the signal

Figure 3. Relative max$^{(4)}$ $M_4^p$ for a Kaplan water turbine$^{(35)}$

Figure 4. Norms of the signal $x^{(4)}$ in the frequency range 10-2000 Hz: (a) Relative max$^{(4)}$ $M_4^p$ calculated for different orders $p$ and $\tau = 3s^{(35)}$; (b) max$^{(4)}$ $M_4^{2.75}$ calculated for different sample times$^{(35)}$

Figure 5. Coefficient of determination $R^2$ calculated as the square of the correlation coefficient between the relative max$^{(4)}$ $M_4^p$ and the knowledge-based cavitation index in a Kaplan water turbine$^{(35)}$
\(x^{(4)}\) provide better results, especially in wider frequency ranges, 10-3000 Hz and 10-4000 Hz \(F(52)\). Non-linear scaling improves the result for \(a = 3\), improving the \(R^2\) value considerably \(F(59)\).

The most sensitive feature to detect strong cavitation at 2 MW is the peak value of \(x^{(4)}\) in the frequency range 10-2000 Hz, but \(x^{(3)}\) can be used as well. The peak values of acceleration signals \(x^{(3)}\) are more sensitive in the range 10-3000 Hz than 10-2000 Hz. The signal \(x^{(4)}\) is also the best choice for observing cavitation at 10 MW and its sensitivity improves when the upper cut-off frequency \(f_H\) is increased from 1000 to 4000 Hz. Acceleration and \(x^{(3)}\) have similar behaviour but somewhat weaker sensitivity. Neither the used signal nor the frequency range seems to have any major influence on the sensitivity to detect cavitation at 59.4 MW. However, \(x^{(4)}\) provides the best sensitivity when \(f_H\) is 3000 or 4000 Hz \(F(52)\).

The sample time \(\tau = 3s\) provided the most sensitive results \(F(53,55,56)\). Ten samples are used to get norms from sufficiently long signals. The order \(p\) of the norm was chosen to be 2.75 and sample time \(\tau = 3s\), which provides a good balance between low and high power ranges since sample time has a strong effect in the low power range (Figure 4(b)). Because of minor calculation requirements this approach, where the generalised norms are obtained from samples, is well suited for intelligent sensors, where analogue derivation can be used to obtain the signal \(x^{(4)}\) from the acceleration signal.

Cavitation indices:

\[
I_{c}^{(k)} = f_{c}^{-1}(\text{relative max}(\|M_{c}^{(k)}\|))
\]

\(k\) are stress indices \(F(4)\), which can be used in power control. High-stress operating conditions should be avoided in order to keep the turbine in good condition. In \(F(59)\), indices were tested in the hot end are very high. Kiln alignment problems are severe because of the high weight affecting the supporting rolls. Lime kilns are essential parts in strongly integrated pulp and paper production.

The distributions of the signals \(x^{(1)}, x^{(3)}\) and \(x^{(4)}\) have been used in monitoring the condition of the supporting rolls of a lime kiln \(F(53,59)\). In this case, the condition index (6) is defined by the standard deviation and five bins of the histogram:

\[
I_{c}^{(k)} = w_i f_i^{-1}\left[\max\left(\|M_{c}^{(k)}\|\right)\right] + \sum_{i=2}^{6} w_i f_i^{-1}(F_{i}^{(k)})
\]

The coefficient vector \(w = [-2\ 1\ 1\ 1 -1 -1\] is based on expertise. The features are \(\sigma_{\alpha} = \frac{\|M_{c}^{(k)}\|}{f_{c}}\) and the fractions \(F_{i}^{(k)}, (k=2) |x^{(i)}| < 2\sigma_{\alpha}, (k=3) 2\sigma_{\alpha} \leq |x^{(i)}| < 3\sigma_{\alpha}, (k=4) 3\sigma_{\alpha} \leq |x^{(i)}| < 4\sigma_{\alpha}, (k=5) 4\sigma_{\alpha} \leq |x^{(i)}| < 5\sigma_{\alpha}, (k=6) |x^{(i)}| \geq 5\sigma_{\alpha}\), where \(a\) is the order of derivation. The fractions \(F_{i}^{(k)}, (k=5\text{ and }6\) are related to faulty situations, and large values for the fractions \(F_{i}^{(k)}, k=2, 3\text{ and }4\) are obtained in normal conditions. Fault situations were detected as a large number of strong impacts. All the features are scaled with functions to \([-2,2]\). The condition index \(I_{c}^{(k)}\) is a number between –2 and 2: high values correspond to good condition and –2 means not allowable condition.

The extended set of data covers surface problems, good conditions after grinding, misalignment after grinding, stronger misalignment, very good conditions after repair work and good conditions one year later. The faulty cases are clearly detected in the new data as well without changing any parameters of the calculation system. This is a very important result \(F(53,58)\). All the supporting rolls can be analysed using the same system. The velocity signal shows hardly any difference between a serious surface problem and an excellent condition.

4.7 Very fast rotating rolling bearings

Detecting bearing faults and unbalance in very fast rotating rolling bearings \(F(60)\) was based on standard deviations calculated for the signal \(x^{(4)}\) on three frequency ranges. The same measurements were used \(F(53)\) to test the norm (3); each calculated value of the norm is divided by the average of the values of the same norm in good conditions. The sample time \(\tau = 3.9\ ms\) corresponds to two rotations when the rotation frequency was 525 Hz. In this case, the condition index (6) is obtained as a weighted sum of the scaled norms of the signal \(x^{(4)}\) calculated for three frequency ranges:

\[
I_{c}^{(k)} = \sum_{i=1}^{4} w_i f_i^{-1}\left[\max\left(\|M_{c}^{(k)}\|\right)\right]
\]

where \(f_i^{-1}\) is the scaling function of the maximum norm (4) in three frequency ranges \(k = 1, 2\text{ and } 3; 10-1000\ Hz, 10-10000\ Hz,\) and 10-50000 Hz. The sequential algorithm introduced \(F(64)\) can be generalised to form:

- Calculating the condition index.
- The condition is normal if \(I_{c}^{(k)} < 1.5\).
- There is an outer race fault in the bearings if \(I_{c}^{(k)} < 0\).
- The condition is unbalance if the standard deviation for the low-frequency range is very high.
- Otherwise the condition is inner race fault in the bearings.
The minimum of the index \( I^{(a)} \) is 2, which is achieved when all the features are at the lowest level and all the weight factors \( w_k = -1/3 \). The weight factors are transformed to obtain the general form of the condition index (6). The faults were detected correctly in (35,50) using the standard deviation \( \sigma_r \).

The applicability of different norms has been studied in (55). Relative max \( \| M^{\alpha} \| \) defined by (4) is shown for three frequency ranges in Figure 6. The reference values are obtained as a mean of the corresponding norms in the normal case. Unbalance can be clearly detected with norms based on all the signals \( x^{(a)} \) in the frequency range 10-1000 Hz. The norms based on the velocity \( x^{(3)} \) have very similar sensitivities for the unbalance and the inner race fault in all frequency ranges. For the acceleration \( x^{(4)} \), the indication of the inner race fault improves considerably with the increasing order \( p \) in the frequency range 10-10000 Hz. Further improvement is achieved when high frequencies up to 50000 Hz are used. The frequency range has a major effect on norms \( \| M^{2} \| \): unbalance can be clearly detected in the low range, the inner race fault in the medium and high range, and finally the outer race fault in the high range. For the outer race fault, the norms \( \| M^{1} \| \) provide a slight indication but are not sufficient for reliable early detection, and higher frequency ranges do not have any effect on the sensitivities. The inner race fault is detected in all studied frequency ranges. In high-frequency ranges, the norms \( \| M^{4} \| \) are at the same level for all three faults if \( p \) is small.

Using the jerk signal \( x^{(4)} \) improves sensitivities considerably: the indication of the inner and outer race faults improves clearly when wider frequency ranges are used. This can be seen especially in the sensitivities of the norms \( \| M^{2} \| \) for the outer race fault. The norms \( \| M^{1} \| \) based on the signal \( x^{(a)} \) provide the best results in all the frequency ranges. The high range has the best indication results: all the sensitivity values are high and the values for different faults are at specific levels. In the low-frequency range, the order \( p \) has a strong effect on the indication of the inner and outer race faults, especially when \( p \) increases from 1 to 3. It is interesting that the signal \( x^{(a)} \) provides better sensitivities for the outer race faults in the range 10-1000 Hz than in the range 10-10000 Hz, if the order \( p \) is high enough. This means that the number of required signal points can be reduced to 1/10 when the signal \( x^{(a)} \) is used.

The absolute mean is sufficient in many cases (Figure 6). Unbalance is detected in the frequency range 10-1000 Hz with all the signals \( x^{(a)} \), \( a = 1, 2, 3 \) and 4. The sensitivities of the rms values \( p = 2 \) and the absolute mean \( p = 1 \) are in most cases quite similar, ie \( \| M^{1} \| \approx \| M^{2} \| \). Considerable differences are only seen in relative max \( \| M^{2} \| \) for the inner and outer race faults. For the inner race fault, a better indicator is the peak value \( x_{\text{peak}}^{(a)} \) obtained as a mean of the highest ten signal values of \( x^{(a)} \) in each sample (Figure 7). When the order \( p \) increases, the sensitivity values tend to increase, almost reaching the peak values \( x_{\text{peak}}^{(a)} \) when \( p = 2.75 \). The \( \| M^{1} \| \) already indicates unbalance with all the values of the order \( p \) (Figures 6 and 7), but the differences from the normal operation increase and become clear for the inner and outer race faults when \( p \) increases. Relative norms \( \| M^{2} \| \) obtained from short samples have considerable fluctuations, especially when high \( p \) values are used (Figure 7). A maximum of these norms obtained from several sequential samples provides more reliable indication: the norms in Figure 6 were obtained from ten samples.

All the faults were detected with the relative max \( \| M^{2} \| \) in the low-frequency range 10-1000 Hz, which is very important for the development of intelligent sensors. In the case studied in (35) different faults were also identified with signals \( x^{(a)} \), \( a = 2, 3 \) and 4. For the norms \( \| M^{2} \| \), the identification of all the faults required the index (10) and the sequential algorithm presented above, ie all the three frequency ranges were used. Also, for the norms \( \| M^{2} \| \), the sequential algorithm would improve the result. For the norms \( \| M^{1} \| \), fault identification also needs the norms obtained from the lower frequency signals. With the norms \( \| M^{1} \| \), all the faults can be identified when high frequencies are included. Generally, identification is more difficult since the severity of the fault has a considerable effect on the features.

The measurement index (7), which combines rms and peak values, provides useful information for the identification of the fault. Good results can be obtained, for example, by using the rms value of velocity and the peak value of signal \( x^{(a)} \), ie \( a_1 = 1 \) and \( a_2 = 4 \) in (7). The orders of derivation \( a_1 \) and \( a_2 \) can also be the same for both the features, as in Figure 7.

The peak values can be replaced with higher order norms and the absolute mean can be used instead of the rms values.
Condition indices (6) can be used if the non-linear scaling is used. To obtain reliable indices, several short samples should be used. Therefore, the results shown in Figure 6 were obtained from ten samples. The MIT index (5) is calculated for different orders $p$ in Figure 8, where the relative maximum norms of the signals $x^{(a)}$, $a = 1, 2, 3$ and 4, are combined by using equal weights $\frac{1}{4}$.

Unbalance and the inner race fault are clearly detected in all these frequency ranges, and also the MIT index for the outer race fault is higher than 3 already in the frequency range 10-1000 Hz if $p > 3.4$. Also, a measurement index which combines the absolute mean of the velocity and a high order norm of $x^{(4)}$ provides a good indication for all three faults in this low-frequency range (Figure 9). Higher frequencies do not necessarily bring additional sensitivity.

5. Discussion

Good sensitivities to faults causing impacts are obtained with generalised central absolute moments and $l_p$ norms if both the orders $a$ and $p$ are sufficiently high (Table 3). Lahdelma has performed the measurements in the test-rigs and in the pulp and paper industry applications discussed above. The sample time $\tau$ connects the features to control applications and frequency ranges are selected on the basis of faults by taking into account the machine or process device under consideration.

5.1 Order of derivation

The sensitivity of the rms and peak values for the faulty gear teeth was clearly improved when the order of derivation increased from 1 to 3 (Table 2). The cavitation analysis provided similar results: $x^{(4)}$ was clearly the best; $x^{(3)}$ provides good indication, which is not sufficient for severity assessment; and $x^{(2)}$ has slightly weaker sensitivity. The fairly insensitive features of $x^{(1)}$ clearly detect only such cavitation that causes low-frequency structural vibrations. For the very fast rotating bearing, $x^{(4)}$ was again the best and $x^{(1)}$ was suitable for detecting unbalance and inner race faults (Figure 6). The sensitivity of $x^{(4)}$ improved considerably when higher frequencies were used. The jerk $x^{(3)}$ is better than $x^{(2)}$ in all the three frequency ranges.

The improvement of sensitivity with the increasing order of derivation is further supported by the features of real order derivatives $x^{(a)}$, obtained for the inner race fault when $a > 2$. Similar results have been obtained for integration in the turbo compressor system: an oil whirl was detected in a sleeve bearing when $a < -2.5$. In both the cases, high sensitivities were achieved in a wide range of $a$ (Figure 1). We can assume that the fault becomes more dangerous if it is detected with features of less sensitive signals.
5.2 Order of moments and norms

In the applications, the rms values had good sensitivities for the oil whirl (Figure 1) and cavitation in the high power range (Figure 3). The peak values were useful for the inner race fault, faulty gear teeth and cavitation in the low power range (Table 3). Both features are needed in order to cover the whole power range in the cavitation analysis. Kurtosis was a good feature for detecting the inner race fault (Figure 1) and cavitation in the low power range (Figure 3).

For impacts, the sensitivity of the moment (2) improves with the increasing order \( p \). For cavitation detection, good results were obtained with moments where \( \alpha = 4, p \approx 4 \) and \( \tau = 3s^{(34)} \). The power range needs to be taken into account, since the level of the moment is very different in the low and high power ranges. As the moment \( \mu^p \) corresponds to the kurtosis, it can be used in the same way as kurtosis.

The \( L_p^p \) norm (3) includes the rms and peak values as special cases \( p = 2 \) and \( p = \infty \), respectively\(^{(1)} \). As the norm (3) asymptotically reaches the peak value when the order increases, a robust feature corresponding to the peak value can be obtained by using a high order, for example \( p = 8 \). This is easier to implement than the mean peak \( x_{mp}^{(50)} \) used in\(^{(50)} \). To make the norm values of different order \( p \) comparable, a relative value of the norm (4) is used in the applications. In the cavitation analysis, the norms were very sensitive to the order \( p \) and the sample time \( \tau \) in the low power range (Figures 3 and 4). In the high power range, considerable differences can only be seen when the order \( p \) is very high. Two specialised features could be combined, but an optimal order can also be selected together with a sample time (Figure 5).

The standard deviations \( \sigma_p \) calculated on three frequency ranges need to be combined for the identification of bearing faults and unbalance in very fast rotating bearings\(^{(34)} \). Even the absolute mean \( (p = 1) \) is sufficient in many cases.
5.3 Sample time

The sample time \( \tau \) connects the features to control applications. The norm values of several samples are combined in each step by selecting the maximum or the mean. This approach was introduced for the moment (2) in(54), but it is used for the norms as well. The sample time is process-specific (Table 3), ranging from 3.9 ms for the very fast rotating bearings of the centrifuge to 15 s, which is now used for the supporting rolls of the lime kiln. The rotation frequency in the centrifuge was more than 6000 times higher than in the supporting rolls. The sample time was 3 s for the water turbine with rotation speed 115 r/min, and 50 ms for the experimental case of the rough surface. The whole signal was used in the earlier applications (Table 3).

5.4 Frequency range

Requirements for the frequency range of the measurements depend on the faults under consideration (Table 3). Unbalance, misalignment, bent shaft, mechanical looseness and some electrical faults, for example, cause vibrations at a relatively low frequency. On the other hand, faults in rolling bearings introduce high-frequency vibrations. Other similar cases are fast rotating gears, cavitation and loose stator coils in electrical motors. The results of the very fast rotating bearings support this, for example higher frequencies are needed for bearing faults (Figure 6). Naturally, higher frequencies do not bring any additional sensitivity to unbalance which occurs in the rotation frequency. The sensitivity of the velocity signal does not increase at all for any of these faults when the upper cut-off frequency was increased above 1000 Hz. This means that \( \sigma_x \) is not very suitable for bearing faults. For acceleration and higher derivatives, the sensitivity on bearing faults usually increases when higher frequencies are used.

In general, appropriate frequency ranges are selected for different signals. One solution in paper machine applications has been presented in(48): displacement is obtained in a low-frequency range, velocity in a slightly higher range up to 1...2 kHz, and acceleration and higher derivatives in up to some kHz. Fairly low-frequency ranges are, in many cases, sufficient for detecting fault-free conditions, for example in a Kaplan water turbine, cavitation-free conditions were reliably detected with features obtained in the frequency range 10-1000 Hz. The strongest cavitation was detected in the frequency range 10-2000 Hz. A detailed analysis requires wider frequency ranges: 10-3000 Hz and 10-4000 Hz(62). The lower limit 10 Hz is used in several international standards of vibration measurement in condition monitoring.

The high sensitivity to faults, which cause impacts, may allow the use of lower frequency ranges, for example the relative \( \max ||FtMf|| \) provided very good results in the low-frequency range. However, the operating conditions of the process should be taken into account when selecting the frequency ranges. All the faults in fast rotating bearings with rotation frequency 525 Hz were detected with features obtained in the frequency range 10-1000 Hz, which is very important for the development of intelligent sensors(55). Sensitivity depends strongly on the frequency range: in this case the sequence is unbalance, inner race and outer race when higher frequencies are used. Even ranges 10-50000 Hz are needed for a more detailed analysis and the identification of faults(54).

The mounting techniques have an influence on the frequency range and dynamic performance of accelerometers. Magnetic mounting may reduce the resonance frequency considerably. To ensure accurate vibration measurements for gear units, the standard ISO 8579:2:1993(E) allows an upper limit of 3 kHz for magnetic mounting(60). Hand-held probes are not acceptable in this case. If high-frequency ranges are not needed, there are no problems of this kind, and hence the repeatability of the measurements is very high, which is a major benefit.

5.5 Condition indices

Several features can be combined with measurement indices (5) and (7). Intelligent condition indices, which take non-linearities into account, work very well for cavitation analysis, the supporting rolls of a lime kiln and very fast rotating bearings. Only one feature was needed in the cavitation index (8). Six features were used in the lime kiln case. For the centrifuge, the identification of all the faults required the index (10) and the sequential algorithm presented above, i.e. all the three frequency ranges were used. Features were divided by the corresponding reference value, but this is not necessary since the scaling functions can equally handle the original feature.

Short sample times and relatively small requirements for frequency ranges make the generalised norms and condition indices feasible for online applications to detect impacts. For \( x^{(4)} \), low-frequency ranges 10-1000 Hz are useful in many cases and, for example, ranges up to 4000 Hz extend applications. The index obtained from \( x^{(4)} \) is the best alternative in many cases, however, the index obtained from \( x^{(3)} \) can also provide good results. Cavitation indices can be used in power control as well.

6. Conclusions

The sensitivity of features to faults causing impacts increases with the order of derivation to some limit, and the signal \( x^{(4)} \) is a good alternative in many applications. The integration of the displacement also provides good results in some applications. Real order derivation and integration introduce additional possibilities. High sensitivities to small faults make generalised moments and norms informative features for early fault diagnosis. Moments need to be normalised in order to obtain dimensionless features. The \( l_p \) norms have the same dimensions as the corresponding signals. The generalised norm can be defined by the order of derivation, the order of the moment and sample time. The low order of derivatives can be compensated for by using higher order moments. Non-linear effects can be taken into account by scaling functions, and several features are combined in condition, measurement and health indices. In some cases only one norm is needed. Methodologies have been developed in experimental systems and several real-world application cases: the scraper of a continuous digester, a washer and a lime kiln in the pulp and paper industry, the gearbox of a sea water pump and a turbo compressor system in an oil refinery, a Kaplan water turbine and a...
centrifuge. Short sample times and relatively small requirements for frequency ranges make this approach feasible for online analysis and control. Cavitation indices, which are examples of stress indices, can be used in power control. High-stress operating conditions should be avoided in order to keep the turbine in good condition. This principle can be extended to other process equipment and machines.

References

7. S Lahdelma, ‘New vibration severity evaluation criteria for condition monitoring’, Research report No 85, University of Oulu, 18 pp, 1992. [In Finnish]