Real Order Derivatives and Generalised Norms in Condition Monitoring with Noisy Data

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Abstract

Rapid changes in acceleration become emphasised upon the derivation of the acceleration signal $x^{(2)}$. Higher order derivatives work very well in the whole range from slowly to very fast rotating rolling bearings. Real order derivatives $x^{(\alpha)}$ provide additional possibilities, and generalised norms with the real order *p* are used in feature extraction. The optimum setting of the orders α and *p* is fault-specific. Added random noise takes the form of additional fault, which in this case makes it more difficult to detect misalignment of a claw clutch by means of acceleration. Derivation reduces the effect of noise by amplifying the higher frequency components of misalignment more than the added noise components. Correspondingly, integration amplifies the lower frequency components. The analysis is fine-tuned with generalised norms. Added noise changes the optimal setting of the orders α and p: high order α combined with high order *p* operates well in misalignment detection, and negative order α combined with low order *p* has good sensitivity for detecting unbalance. The results clearly show that an extended analysis with a wide range of orders α and *p* is needed for the detection of simultaneous faults in order to obtain the best sensitivity for specific faults, also if the measurements are noisy. The power of generalised norms is in selecting the amplified frequency ranges by the order of derivation and in fine-tuning the sensitivity with the order of moment.

Keywords: Condition monitoring, real and higher order derivatives, fractional derivatives, feature extraction, generalised norms, noise reduction

1. Introduction

Any attempt to detect different types of machine faults reliably at an early stage requires the development of improved signal processing methods. The mathematical background of fractional derivatives has a very long history that goes back to Leibniz, Johann Bernoulli and L'Hospital. The theory of signal processing and ideas for practical problems and more about the history of fractional integrals and derivatives can be found in (1). Practical condition monitoring application use vibration measurements, which were started by means of mechanical or optical instruments, used displacement

$x = x(t)$. The next step was the adoption of velocity i.e. $x^{(1)}$ signals, which were obtained either by the derivation of displacement or using sensors whose output was directly $x^{(1)}$. Unbalance and misalignment can be detected successfully with the signals x and $x^{(1)}$, but they do not usually have sensitivity enough to allow the detection of impact-like faults at a sufficiently early stage. Nowadays measurements are performed more frequently with accelerometers, and the signals x and $x^{(1)}$ are obtained from the $x^{(2)}$ signal through analogue or numerical integration. [2,3]

Higher order derivatives introduced in [4,5] provide additional methods for vibration analysis [3]. The first time derivative of acceleration, known as jerk, has been used for assessing the comfort of travelling, e.g. in designing lifts, and for slowly rotating rolling bearings [6], and early practical studies [5,7] confirmed this: even better results were obtained with $x^{(4)}$, later known as napse [1]. The changes in acceleration are rapid and become emphasised upon the derivation of the signal $x^{(2)}$. The integration of displacement has been introduced in [4]. Fractional integrals and derivatives are discussed in [8], and different approaches have been reviewed in [3].

Statistical analysis provides various features for the signals: expectation and moments are used in developing features [1]. The central value alternatives are mean, median and mode. In vibration analysis, root mean square (rms) and peak values are the most commonly used features [9]. Dimensionless features are obtained by normalisation. Normalised moments, skewness and kurtosis, are widely used special cases corresponding to orders three and four, respectively. Probability distributions can be taken into account. If all the signal values are positive, also the norms generalised to any real-valued order produce real-valued features. To obtain dimensionless features, these norms must be normalised. A generalised central moment introduced in [10] works well even with short sample times. The generalised norms introduced in [9] have same dimensions as the signals to be analysed.

Sufficiently sensitive and robust features are necessary in more detailed analysis. Vibration indices based on several higher derivatives in different frequency ranges were already introduced in 1992 [4]. Operating conditions can be detected with linguistic equation (LE) approach, where nonlinear scaling is used to extract the meanings of variables from measurement signals, was introduced in 1991 [11]. The combined approach has been summarised in [3]. A limit of sensitivity is reached on a certain order of derivation, and the optimal sensitivity is chosen by the order of the moment. Both the orders can be chosen fairly flexibly from the optimal area. Sample time, which connects the features to the control applications, is process-specific. The analysis methods allow the use of lower frequency ranges, and a multisensor approach can also be used. The approach suits well for rotating process equipment with speed ranging from very slow to very fast. [12]

Test rigs are flexible systems in developing methods for fault diagnosis by using signal processing [13], feature extraction [14,15], advanced modelling [14], and tuning methods [15,16]. Excellent classification results have been achieved with a reduced set of sensors [17]. Previous studies with a test rig [18] provided promising possibilities to handle the effects of added noise with derivation and integration combined with generalised norms.

This paper addresses the real order derivatives and generalised norms in analysing vibration measurements obtained from a test rig with special emphasis on effects of the noise on the sensitivity of the features.

2. Problem statement

In this paper, a test rig, which consists of an electric motor and a transmission between two axes with roller bearings [14], has been used (Figure 1). The primary function is to simulate different fault modes that arise when defective elements are added to the rig. The test rig has been built by PIM Bt. and later modified in Mechatronics and Machine Diagnostic Laboratory. It is a convertible small size device with 0.18 kW AC motor. All measurements were done by the Mechatronics and Machine Diagnostic Laboratory, Department of Mechanical Engineering in University of Oulu, see [19].

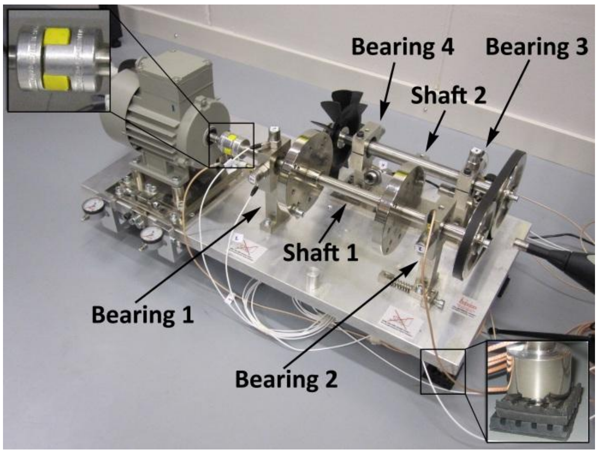


Figure 1. Test rig.

3. Signal Processing and feature extraction

Feature extraction is based on velocity $x^{(1)}$, acceleration $x^{(2)}$ and higher derivatives, $x^{(3)}$ and $x^{(4)}$, and real order derivatives $x^{(\alpha)}, \alpha \in R$. The other signals have been obtained from acceleration through analogue [20] or numerical integration and derivation [21].

3.1 Derivation and integration

Let us examine functions of the form

$$x(t) = Xe^{i\omega t}, \qquad (1)$$

where $0 < \omega \in R$ and $X \in R$ are constants, t is a real variable, and $i = \sqrt{-1}$. In 1997 [22], Lahdelma defined a real order derivative $x^{(\alpha)}$ of the function (1) as

$$x^{(\alpha)} = \omega^{\alpha} X e^{i(\omega t + \alpha \frac{\pi}{2})}, \qquad (2)$$

which means that $x^{(\alpha)} = (i\omega)^{\alpha} x$.

The calculation of the time domain signal $x^{(\alpha)}(t)$, which is based on a rigorous mathematic theory [8], is performed with three steps. The fast Fourier transform (FFT) is used for the displacement signal $x(t)$ to obtain the complex components $\{X_k\}, k = 0,1,2,...,(N-1)$. The corresponding components of the derivative $x^{(\alpha)}(t)$ are calculated as follows:

$$X_{\alpha k} = (i\omega_k)^{\alpha} X_k . \quad (3)$$

Finally, the resulting sequence is transformed with the inverse Fourier transform FFT $^{-1}$, which produces the signal $x^{(\alpha)}(t)$. Since the vibration analysis is now based on the acceleration signals, the components of the derivative are obtained with

$$X_{\alpha k} = (i\omega_k)^{\alpha-2} X_{2k} , \quad (4)$$

where the complex components $\{X_{2k}\}$ are calculated from the acceleration signal $x^{(2)}$. The fast Fourier transform is explained in [23]. To obtain complex derivatives we use $z = \alpha + \beta i$ instead of α. [24] Alternatively, the derivatives can be calculated in time domain [8], but the method presented above is more straightforward.

Derivation and integration can be performed with both an analogue and a digital technique. In the applications discussed in [20,25], acceleration signals have been recorded in the frequency range from 10 Hz to 20 kHz. The linear range of the analogue differentiator/integrator was from 2 to 2000 Hz. The equipment had a low pass filter with a cut-off frequency of 2000 Hz. Sharp bandpass filtering was applied to the analogue velocity signal whose frequency range was from 10 or 100 Hz to 1000 Hz. After filtering to the linear range, the resulting signals were transferred to the computer by means of a data acquisition card. [20] In the digital approach, the recorded acceleration signals are transferred to the computer and derivated or integrated numerically with LabVIEW, and all the signals were filtered by means of a sixth order Butterworth bandpass filter [26]. A combined analogue and digital technique can markedly reduce the amount of data. This is important in intelligent sensors, especially if the rotation speed is very low.

3.2 Generalised moments and norms

The mathematical expectation, expected value, or briefly the expectation, of a random variable is a very important concept in probability and statistics. In condition monitoring, the absolute values of the dynamic part of the signals are used to introduce features, which are sufficient to detect faults at an early stage [1]. The generalised central absolute moment about zero can be normalised by means of the standard deviation σ_α of the signal $x^{(\alpha)}$:

$$ {}^{\tau}_{\sigma}M^{p}_{\alpha} = \frac{1}{N(\sigma_\alpha)^p}\sum_{i=1}^{N}\left|x_i^{(\alpha)}\right|^p = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_i^{(\alpha)}}{\sigma_\alpha}\right|^p , \quad (5)$$

which was presented in [10]. The real number α is the order of derivation, and the real number p is the order of the moment. The moment is obtained from the absolute values of signals $x^{(\alpha)}$. The signal is measured continuously, and the analysis is based on consecutive equally sized samples. Duration of each sample is called sample time, denoted by τ. The number of signal values $N = \tau N_s$ where N_S is the number of signal values which are taken in a second. The peaks of the signal have a strong effect on the moment (5), which can be used in the same way as kurtosis [26]. The moments calculated for higher order derivatives are more sensitive to impacts than the ones calculated for velocity. The sensitivity of the moment improves when the order p of the moment increases. [1]

The generalised norm of $x^{(\alpha)}$ defined by

$$\left\| x^{(\alpha)} \right\|_p \equiv \left\| {}^{\tau}M_{\alpha}^{p} \right\|_p = ({}^{\tau}M_{\alpha}^{p})^{1/p} = \left(\frac{1}{N}\sum_{i=1}^{N}\left| x_i^{(\alpha)} \right|^p\right)^{1/p}, \qquad (6)$$

where $p \in R$, has the same dimension as the corresponding signal $x^{(\alpha)}$. This norm combines two trends: a strong increase caused by the power p and a decrease with the power $1/p$. The norm (6) include the norms from the minimum to the maximum, which correspond the orders $p = -\infty$ and $p = \infty$, respectively [1]. The norm (6) includes the absolute mean ($p = 1$), and the root mean square (rms) value ($p = 2$) as special cases. The norms can be obtained as the norm for the norms of individual samples. A feature can also be defined as a maximum of the norms $({}^{\tau}M_{\alpha}^{p})_i^{1/p}$ calculated from different samples $i = 1,..., K_S$, i.e.

$$\max(\left\| {}^{\tau}M_{\alpha}^{p} \right\|_p) \equiv \max_{i=1,\ldots,K_S} \left\{ ({}^{\tau}M_{\alpha}^{p})_i^{1/p} \right\}. \qquad (7)$$

The number of signal values in each sample is equal and defined by the sample time and the number of signal values in a second. The sample time τ is an essential parameter in the calculation of moments and norms. [1,12] The sensitivity of the moment improves when the order p of the moment increases. The order p is selected to be close to four, and the sample time should be fairly short in the cavitation analysis.

The kurtosis and the crest factor are dimensionless features, which combine features of two orders: the kurtosis defined by (5) is the fourth moment normalised with the second order norm, i.e. $p_1 = 4$ and $p_2 = 2$, and the crest factor or peak-to-average ratio is the peak value normalised with the rms value $(p_2 = 2)$. The peak value corresponds to a high order norm $(p_1 = \infty)$ or the maximum of the absolute values.

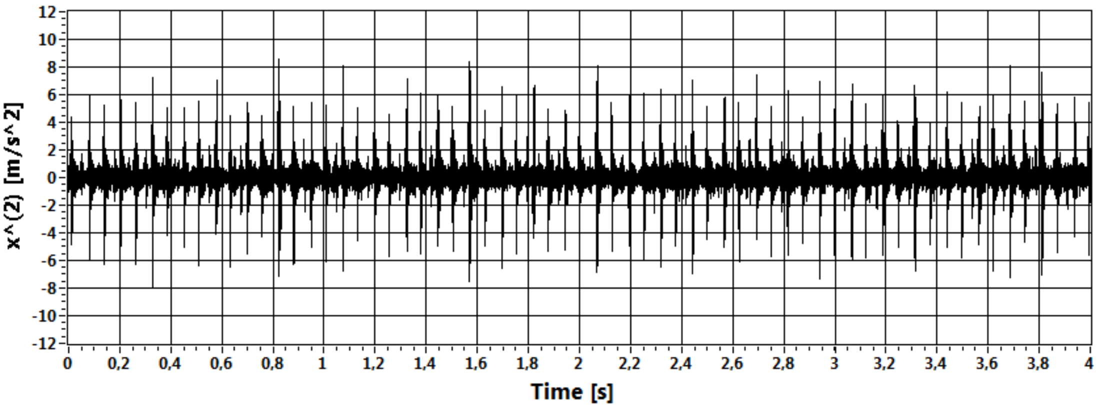
The signal can be divided into short samples and features calculated for these samples. The rms values or peak values of several samples are then combined in fault detection, e.g. by calculating an average from the highest three values of the sample features. The sample time τ is an essential parameter in the calculation of moments and norms. Short sample times were found to be good in [10]. However, sufficiently long signals are required to produce reliable maximum moments and material for analysing short-term cavitation.

4. Analysis of noisy signals

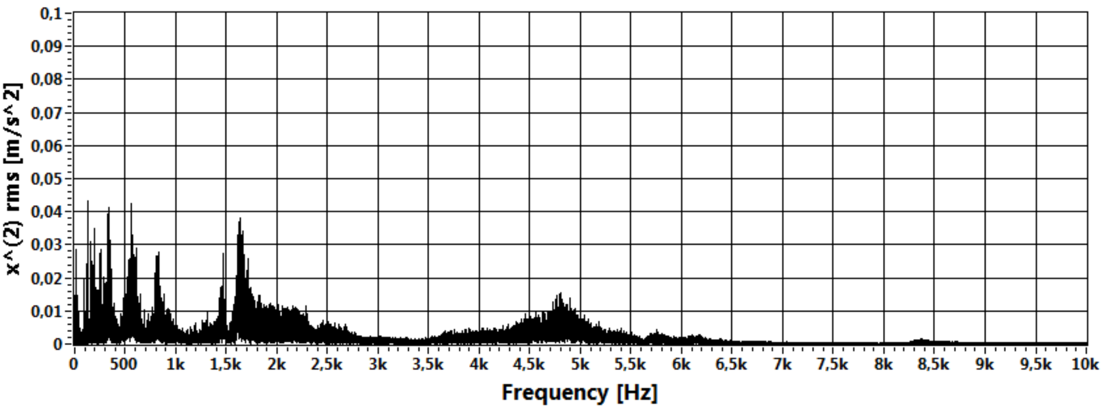
4.1 Measurements

Acceleration signals were selected from the measurements carried out on the test rig shown in Figure 1. The misalignment faults were introduced by moving the motor in the horizontal plane. The rotor unbalance was on the drive shaft. Only one horizontal accelerometer of the bearing 1 was investigated, because the vibration caused by misalignment is clearly the strongest in this measurement point. The acceleration signal with a sample length of 4 s was used in the calculation of features and the rms amplitude spectra. Signal processing, such as derivation and filtering, was performed in the frequency domain and an ideal filter was used. The frequency range was from 3 Hz to 10 kHz. [19]

The measurements were carried out for both the faulty cases and corresponding intact cases. One case, where the rotation frequency was 8 Hz, was chosen for analysing the effect of additional noise. The faulty case had two very clear faults combined: the misalignment was 0.35 mm and the unbalance 11 g (552 gmm). The acceleration signals and their spectra are shown in Figures 2 and 3.

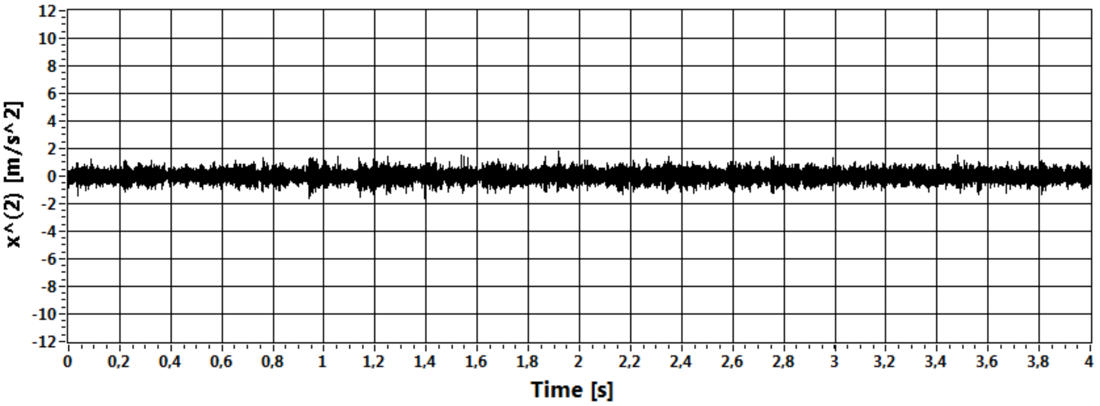


(a) Acceleration signal.

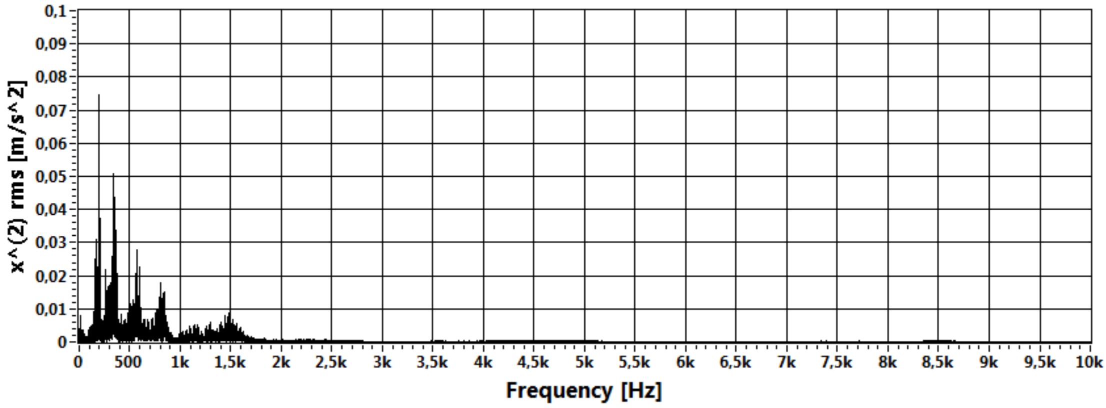


(b) The rms amplitude spectrum.

Figure 2. Acceleration signal obtained from the faulty case.



(a) Acceleration signal.



(b) The rms amplitude spectrum.

Figure 3. Acceleration signal obtained from the intact case.

To test the robustness of the analysis methods, Gaussian noise with standard deviation 5 m/s^2 was added to the both acceleration signals in frequency range from 50 to 3000 Hz. The signal obtained from the intact case (Figure 3) is hidden by the noise (Figure 4). Only the strongest peaks can be seen in the spectrum (Figure 5).

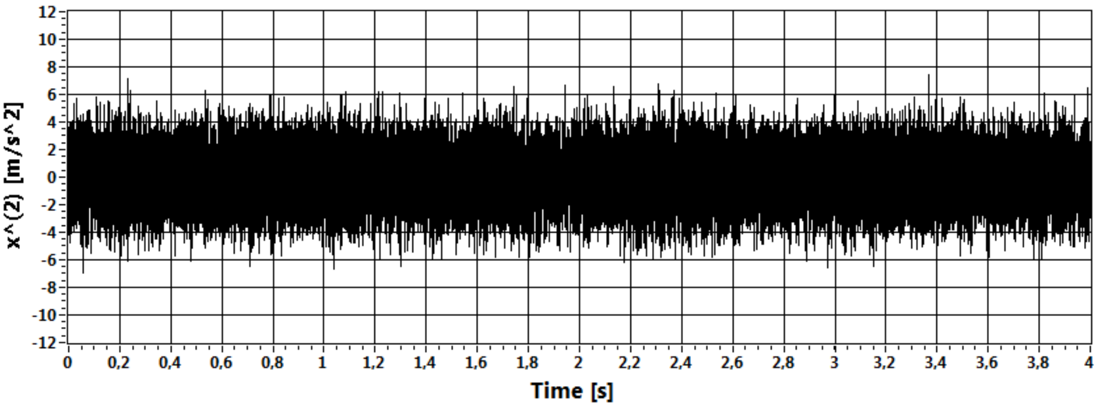


Figure 4. Acceleration signal of the intact case added with noise.

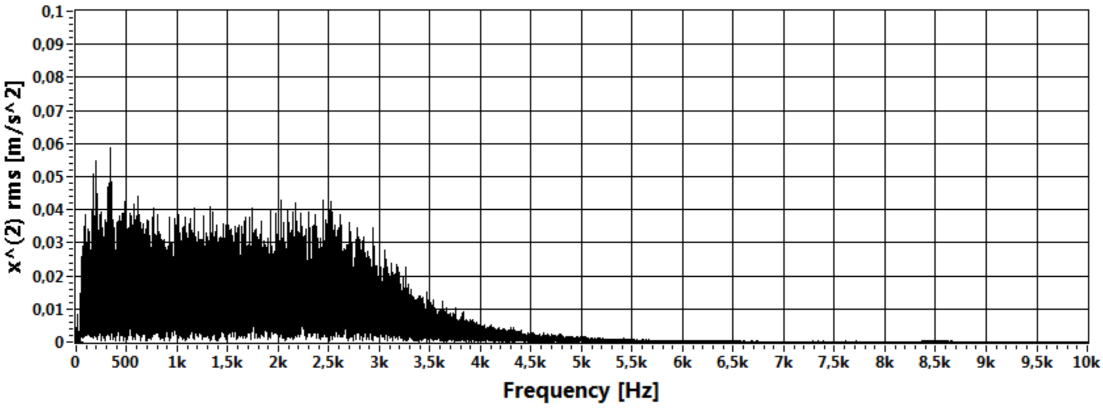


Figure 5. The rms amplitude spectrum of the noisy acceleration signal of the intact case.

Although, some impacts are seen in the signal (Figure 6), the noise masks the fault structures which are evident in the original acceleration signal (Figure 2 a). Only the signal components in the range from 4 to 5.5 kHz can be seen in the spectrum (Figure 7), since they are clearly above the frequency range of the added noise.

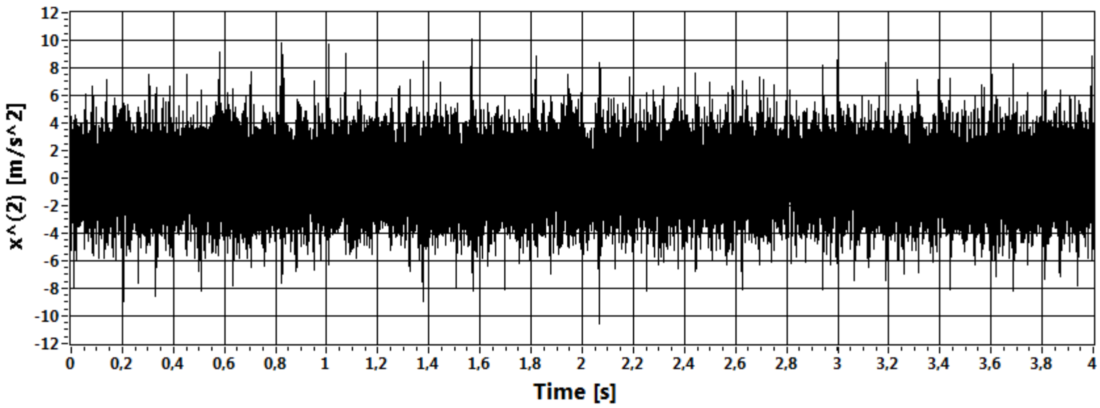


Figure 6. Acceleration signal of the faulty case added with noise.

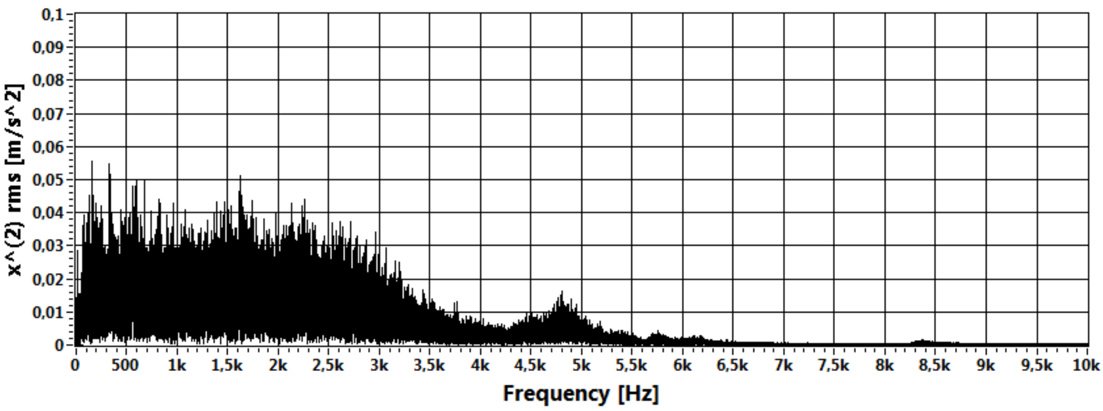
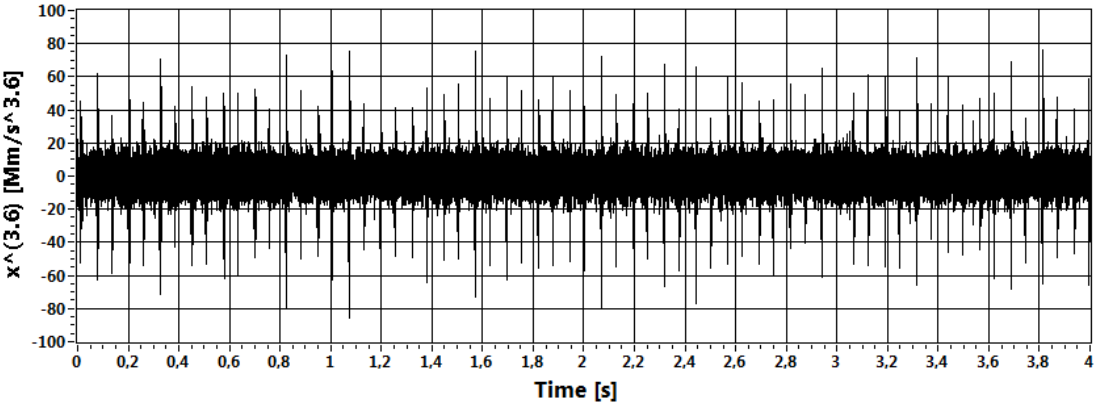


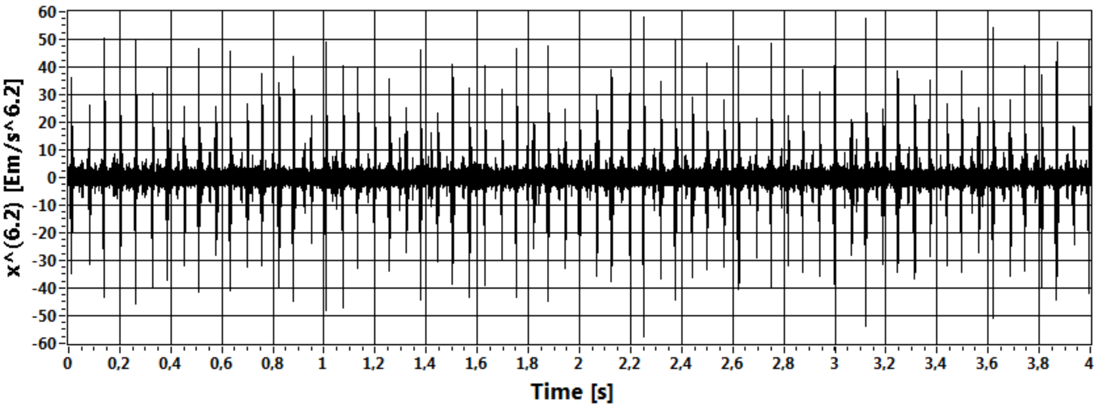
Figure 7. The rms amplitude spectrum of the noisy acceleration signal of the faulty case.

4.2 Positive order derivatives

Higher order derivatives are needed to detect the impacts caused by the misalignment. The signals of orders 0.2, 0.4, ... 1.8, 2.2, 2.4, ... 10 were calculated from the measured acceleration signals. The impacts become clearly visible in the signal $x^{(3.6)}$ (Figure 8 a) and very clear in the signal $x^{(6.2)}$ (Figure 8 b). Derivation reduces the effect of noise by amplifying higher frequency components from misalignment more than the added noise components (Figure 9).



(a) Signal $x^{(3.6)}$.



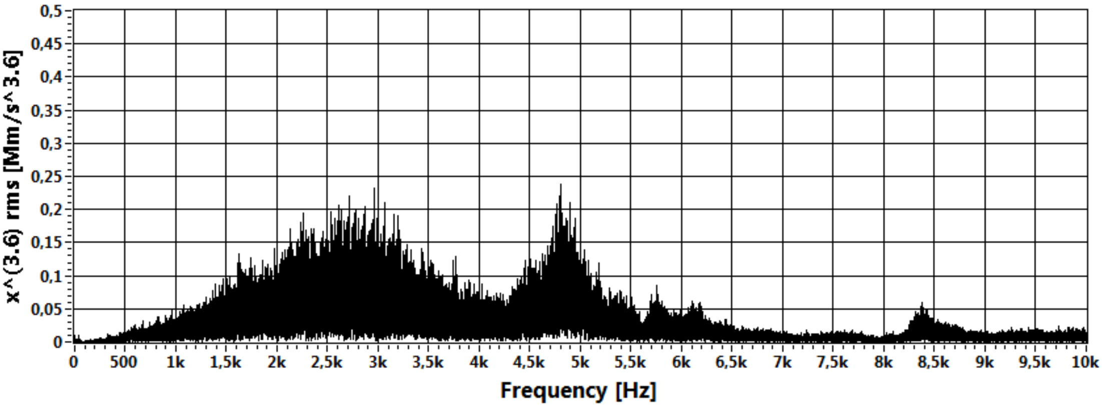
(b) Signal $x^{(6.2)}$.

Figure 8. The signals $x^{(\alpha)}$ obtained from the noisy acceleration.

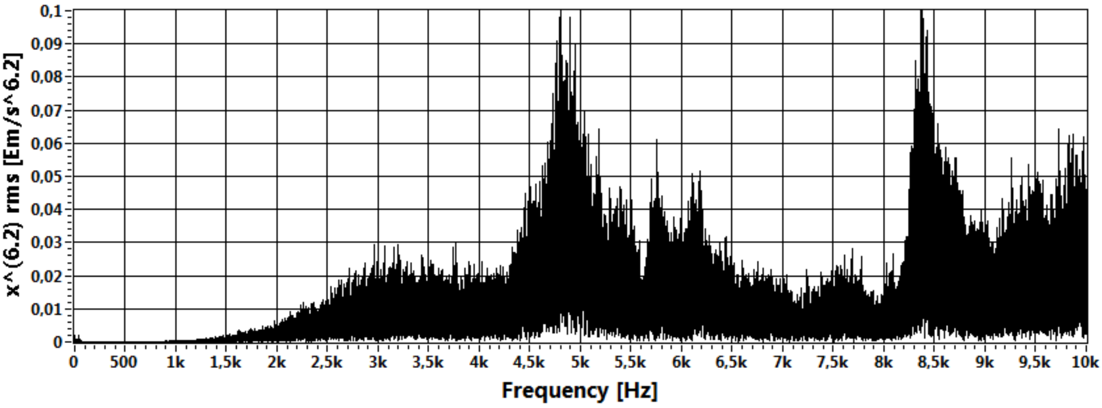
For the original acceleration signal, the impacts can be seen best when the order of derivation is 3.6, and the sensitivity starts to decrease when the order further increases. The order is in the same range as in several earlier studies where detecting impacts is important. In a lime kiln [27], misalignment was detected with the order 4.25. The optimal order was 4 in many applications: faulty bearings in a digester, a very slowly rotating bearing in a washer, a water turbine and a very fast rotating rolling bearing. Even for a scratch on a rough surface, the order was 3.75. [12]

Higher order derivatives are needed for analysing the noisy acceleration signal, since the frequency range of the added noise has a clear effect on the spectrum of the signal

$x^{(3.6)}$ shown in Figure 9 a. The effect is considerably reduced for the signal $x^{(6.2)}$, see Figure 9 b.



(a) The spectrum of the signal $x^{(3.6)}$.



(b) The spectrum of the signal $x^{(6.2)}$.

Figure 9. The rms amplitude spectra of the signals $x^{(\alpha)}$ obtained from the noisy acceleration.

4.3 Negative order derivatives

The unbalance is indicated well with the displacement and velocity [12]. The signals of order -2, -1.8, ... -0.2 were calculated from the measured acceleration signals. In this case, the displacement signal $(\alpha = 0)$ still includes some impacts (Figure 10), which are removed by continuing the derivation to the order -0.6 (Figure 11). The rotation frequency 8 Hz is clearly seen in both signals. This frequency is well below the frequency range of the noise, 50 – 3000 Hz.

Displacement obtained from the original acceleration signal operates better than the corresponding velocity, but differences between the signals of lower order are very small. Unbalance of the very fast rotating bearings was detected in [28] with all the signals $x^{(\alpha)}$, $\alpha = 1, 2, 3, 4$, in the frequency range 10-1000 Hz.

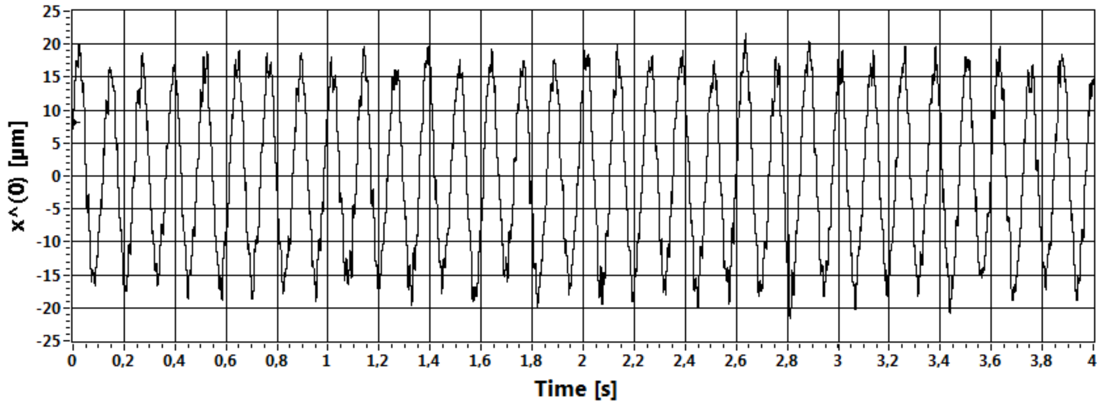


Figure 10. Displacement signal $x^{(0)}$ obtained from the noisy acceleration.

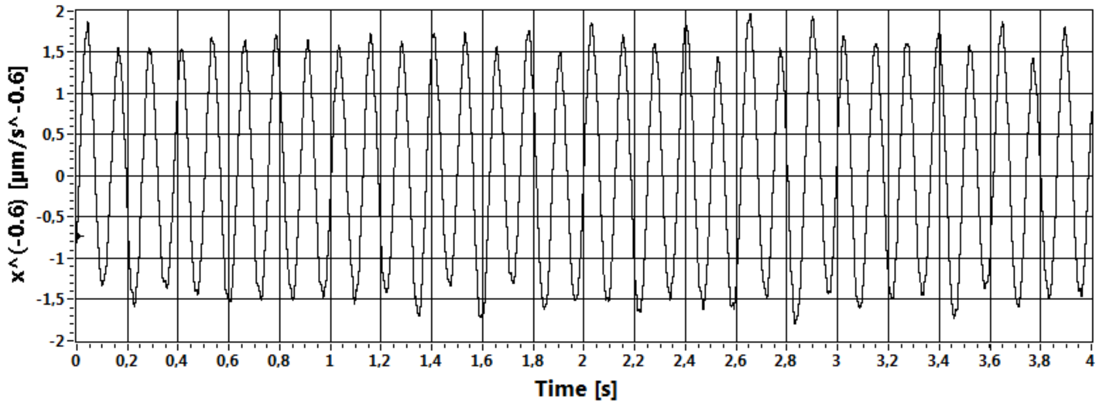


Figure 11. Signal $x^{(-0.6)}$ obtained from the noisy acceleration.

4.4 Feature extraction

The generalised norms with orders p from 0.2 to 8 with a step of 0.2 were calculated for all the signals $x^{(\alpha)}$, $\alpha = -2, -1.8, \ldots 10$. The whole sample was used, i.e. the sample time $\tau = 4$ seconds in this study. Sensitivity is defined by dividing the norm of the signal in the faulty case by the norm in the non-faulty case.

For the original signals, high sensitivity for misalignment is achieved when $\alpha \approx 3.6$ and $p \geq 5$. For the order α, this area is quite narrow, but the order p can be chosen from a wide range (Figure 12). The unbalance is detected well when $\alpha < 1$, and low orders p are slightly better than high orders for this fault.

For the signals with added noise, the higher orders are needed to get high sensitivity for misalignment: a good solution is to use $\alpha = 6.2$ and $p \geq 5$. For the order α, this area is quite narrow, but the order p can be chosen from a wide range (Figure 13). Unbalance is detected well when $\alpha < 1$, and low orders p are slightly better than high orders for this fault. For this case the best order $p = -0.6$.

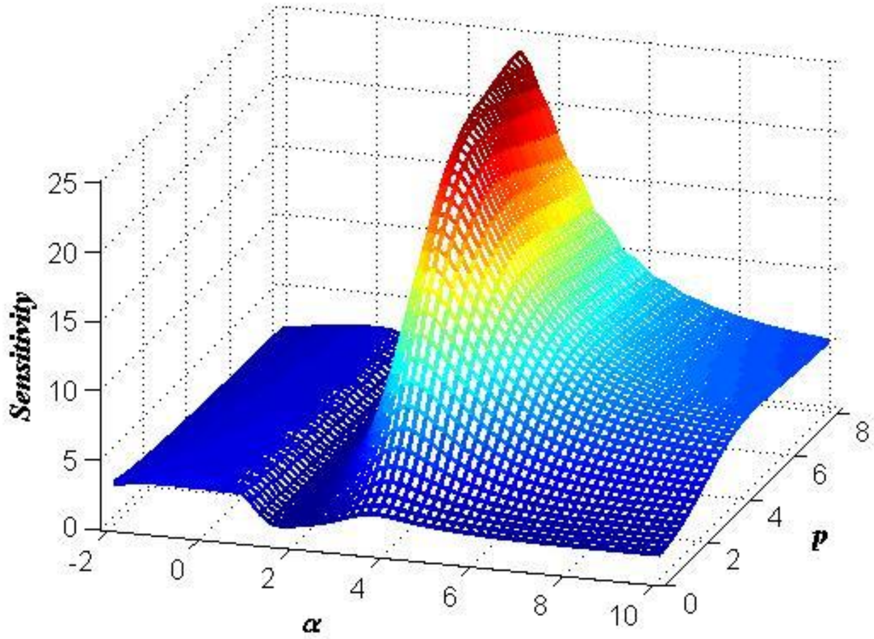


Figure 12. Sensitivity of norms obtained from the signals in the faulty case without additional noise.

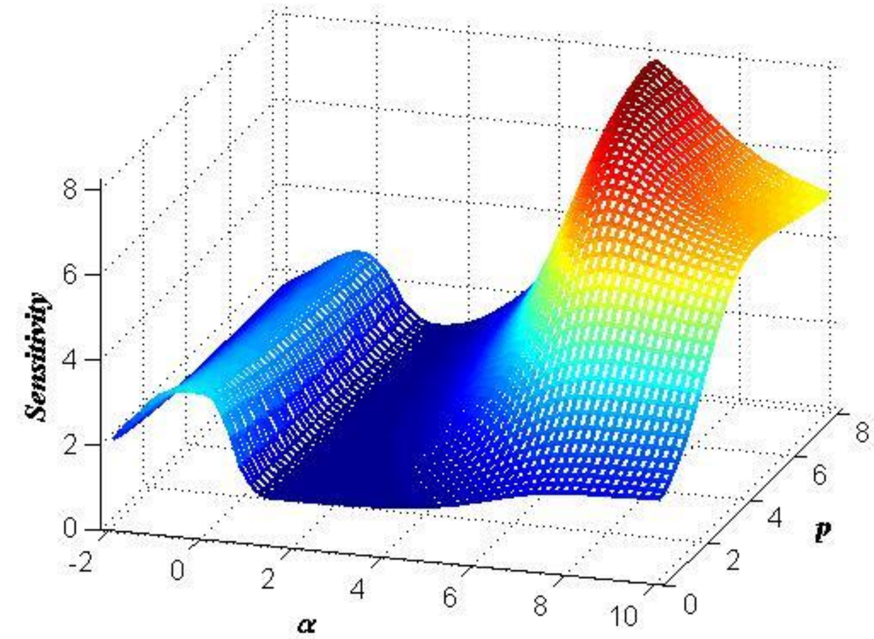
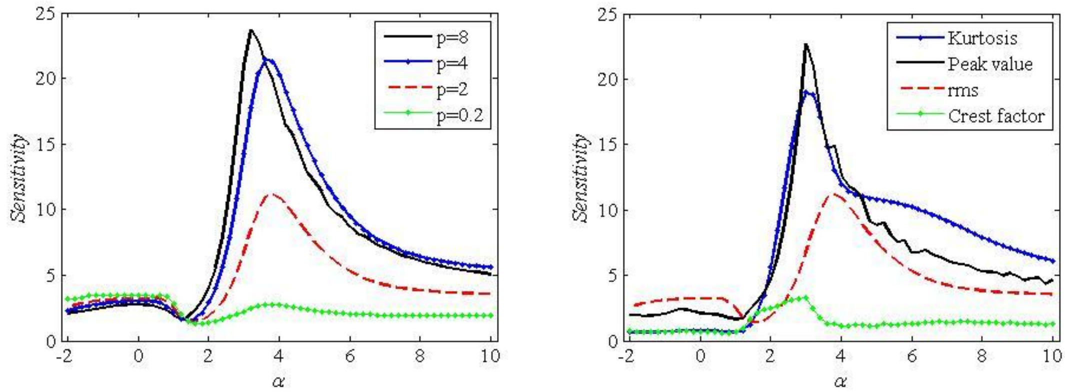


Figure 13. Sensitivity of norms obtained from the signals in the faulty case with additional noise.

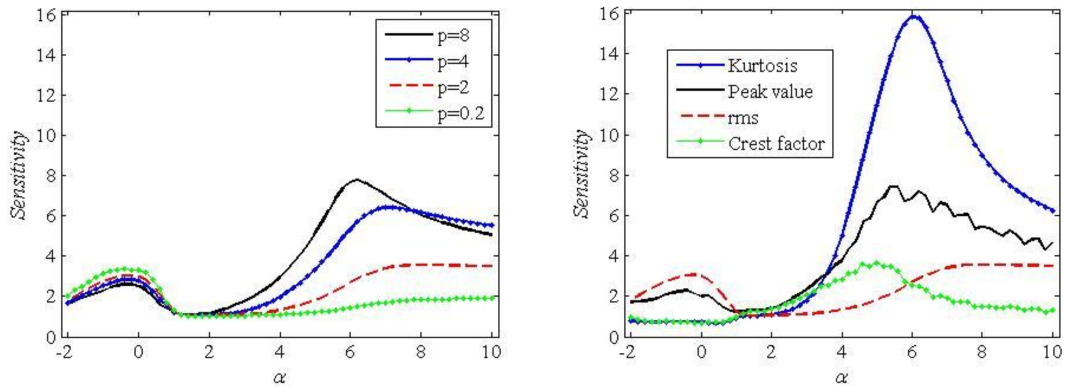
For the original signals, high order norms provide the best sensitivity for misalignment if the order of derivation is in the optimal range (Figure 14 a). Also the rms value has high sensitivity. For unbalance, the sequence is opposite but with small differences compared to the values in higher orders. Peak value and kurtosis have high sensitivities in the area when $\alpha \approx 3.6$ (Figure 14 b). Crest factor has some kind of sensitivity only when $\alpha \approx 2 \ldots 3$.



(a) Generalised norms (b) Other statistical features.

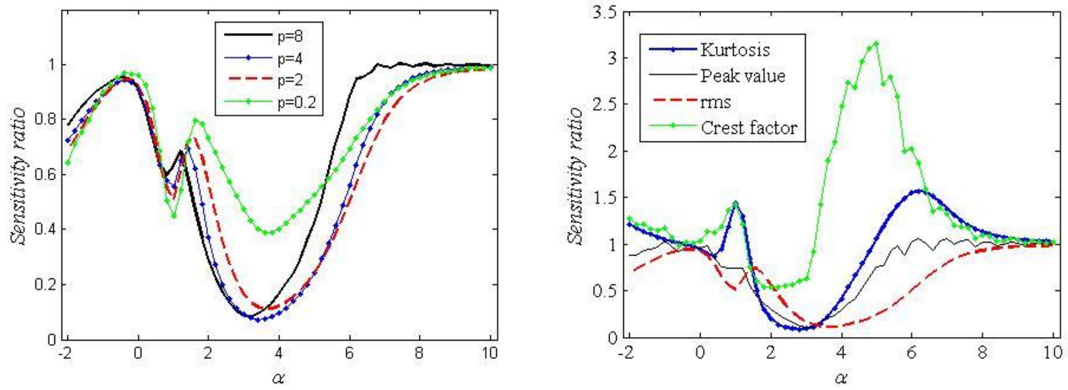
Figure 14. Features obtained from the signals in the faulty case without additional noise.

Sensitivities of generalised norms decrease in a wide range of orders when noise is added to the acceleration signal (Figure 15 a). For high orders $\alpha \approx 3.6$ and low orders around -0.6 there are not much changes. High order norms suit for detecting misalignment and low order norms for detecting unbalance. The calculated sensitivity of kurtosis is very high in some ranges of α, even higher than values obtained for the original signal (Figure 15 b). Sensitivity ratios are reasonable for the generalised norms (Figure 16 a), but the ratios for kurtosis and crest factor are higher than one, which is not acceptable (Figure 16 b). Low rms values give rise to kurtosis when the fourth moment increases. The rise is even higher for the crest factor. The sensitivity of kurtosis is affected by the sensitivities of rms values and $\left\| {}^{4}M_{\alpha}^{4} \right\|_4$ (Figure 17). Sensitivities of peak values and $\left\| {}^{4}M_{\alpha}^{8} \right\|_8$ are very similar (Figure 18).



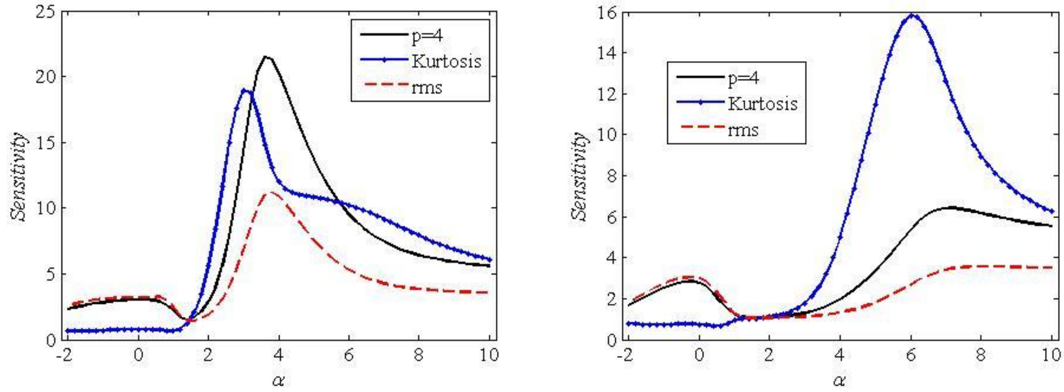
(a) Generalised norms (b) Other statistical features.

Figure 15. Features obtained from the signals in the faulty case with additional noise.



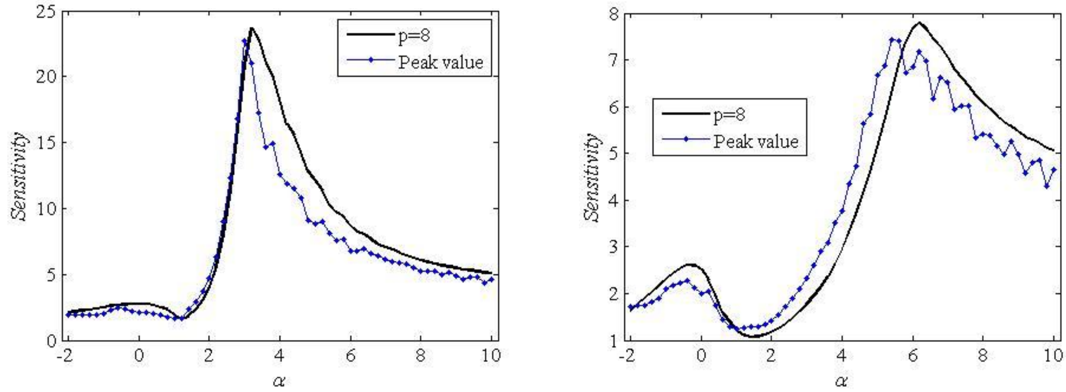
(a) Generalised norms (b) Other statistical features.

Figure 16. Sensitivity ratio of the noisy and original signals for features.



(a) Signals without additional noise. (b) Signals with additional noise.

Figure 17. Feature comparisons: kurtosis, rms and $\left\| {}^{4}M_{\alpha}^{4} \right\|_{4}$.



(a) Signals without additional noise (b) Signals with additional noise.

Figure 18. Features comparisons: peak value and $\left\| {}^{4}M_{\alpha}^{8} \right\|_{8}$.

Generalised norms $\left\| {}^{\tau}M_{\alpha}^{p} \right\|_{p}$ provide high sensitivity for detecting misalignment and unbalance from noisy acceleration measurements. The smallest sensitivity decrease is achieved in the high and low orders of derivation, and the high range of orders is further extended by using high order norms (Figure 19). The optimal setting of the orders α and p changes: (1) high order α combined with high order p operates well in misalignment detection and (2) negative order α combined with low order p has good sensitivity for detecting unbalance (Figure 13). Peak values can be replaced by a high order norm, e.g. $\left\| {}^{4}M_{\alpha}^{8} \right\|_{8}$.

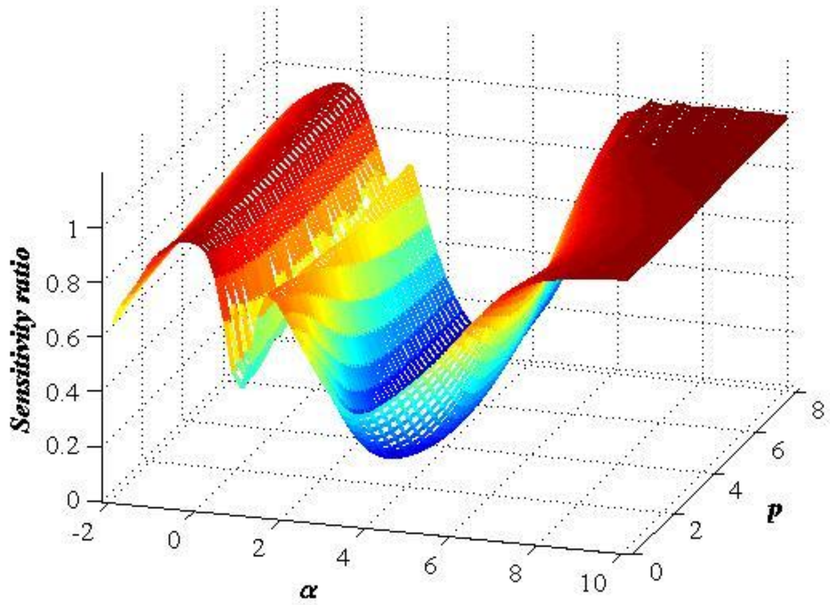


Figure 19. Sensitivity ratio of the noisy and the original signals for generalised norms $\left\| {}^{\tau}M_{\alpha}^{p} \right\|_{p}$.

The results clearly show that an extended analysis with a wide range of orders α and p is needed for the detection of simultaneous faults in order to obtain the best sensitivity for specific faults, also if the measurements are noisy. In practice, selected frequency ranges are amplified by the order of derivation, and selected amplitude ranges are emphasised with the order of moment.

5. Conclusions

Generalised norms provide informative features for diagnosing simultaneous faults even when noisy measurements are used. The extended analysis with wide ranges of orders α and p reveals optimal ranges for detecting misalignment and unbalance. The power of generalised norms is in selecting the amplified frequency ranges by the order of derivation and in fine-tuning the sensitivity with the order of moment.

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