

Intelligent Stress Indices in Fatigue Detection

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Abstract

Fatigue is caused by repeated loading and unloading. The mechanism proceeds through cracks formed when the load exceeds certain thresholds. Structures fracture suddenly when a crack reaches a critical size. Intelligent stress indices based on nonlinear scaling provide good indicators of the severity of the load. The stress index I_s is -2 when the stress is negligible, and levels $\{-1, 0, 1\}$ are analogue to the lower limits of the vibration severity ranges {usable, still acceptable, not acceptable} defined in the VDI 2056. A Wöhler curve is represented by a linguistic equation (LE) model, $I_s = \log_{10}(N_c)$, where the stress index can be a scaled value of stress or a scaled value of a generalised norm obtained from vibration signals. The scaling of the logarithmic values of the number of cycles is linear. As the LE model is nonlinear, it covers a wide operating range. The system may also contain several specific equations corresponding different operating point, e.g. low, normal and high stress. The analysis of material fatigue can be based on existing Wöhler curves or on test results. For process equipment, the curves are gradually refined by recursive tuning of the scaling functions. The contribution of the stress is calculated in each sample time, which is taken as a fraction of the cycle time. The previous history can be updated whenever the scaling functions are changed. The cumulative sum of the contributions indicates the deterioration of the condition, and the simulated sums can be used for predicting the failure time. The high stress contributions dominate in the summation. The summation of the contributions also reveals repeated loading and unloading, and the individual contributions provide indications of the severity of the effect. The approach also operates well when the load is complex, and random load can be handled with simulation.

Keywords: Fatigue detection, nonlinear scaling, intelligent stress indices, vibration analysis, condition monitoring

1. Introduction

The history of fatigue analysis begins already in 1837, when Wilhelm Albert published in Clausthal the first fatigue-test results ^(Schutz96). After systematic tests Wöhler concludes that cyclic stress range is more important than peak stress and introduces the concept of endurance limit. Fatigue is a progressive and localised structural damage that is caused by repeated loading and unloading. The nominal maximum stress values are

less than the ultimate tensile stress limit, and may be below the yield stress limit of the material. The mechanism proceeds through cracks formed when the load exceeds certain thresholds. Structures fracture suddenly when a crack reaches a critical size. The shape of the structure will significantly affect the fatigue life; square holes or sharp corners will lead to elevated local stresses where fatigue cracks can initiate. Round holes and smooth transitions or fillets are therefore important to increase the fatigue strength of the structure.

Advanced signal processing methods and intelligent fault diagnosis have been developed to detect different types of machine faults reliably at an early stage. Dimensionless indices, which are obtained by comparing each feature value with the corresponding value in normal operation, provide useful information on different faults, and even more sensitive solutions can be obtained by selecting suitable features.^(Insight07) Generalised moments and norms include many well-known statistical features as special cases and provide compact new features capable of detecting faulty situations (CM08Signal,IJCM11a,IJCM11b). Intelligent models extend the idea of dimensionless indices to nonlinear systems. Operating conditions can be detected by means of a Case-Based Reasoning (CBR) type application with linguistic equation (LE) models and fuzzy logic (SafeProcess94,Juuso99,Juuso04IJA). The basic idea is nonlinear scaling, which was developed to extract the meanings of variables from measurement signals^(IFAC92).

This paper addresses fatigue prediction by using intelligent stress indices, which are based on advanced signal processing, generalised norms and nonlinear scaling.

2. Fatigue

ASTM International, earlier known as the American Society for Testing and Materials, defines fatigue life, N_f , as the number of stress cycles of a specified character that a specimen sustains before failure of a specified nature occurs.^(StephensFuchs01) In high-cycle fatigue situations, materials performance is commonly characterized by an S-N curve, also known as a Wöhler curve (Figure 1). This is a graph of the magnitude of a cyclic stress (S) against the logarithmic scale of cycles to failure (N). These curves are material specific^(MarinesBB01).

S-N curves are derived from tests on samples of the material where a regular sinusoidal stress is applied by a testing machine which also counts the number of cycles to failure. Probability distributions that are common in data analysis and in design against fatigue include the lognormal distribution, extreme value distribution, Birnbaum–Saunders distribution, and Weibull distribution.

In practice, the sequence of load is complex, often random, including large and small loads. Rainflow analysis and histograms of the cyclic stress are used to assess the safe life in these cases. Effects of each stress level are taken into account in calculations of cumulative damage. Individual contributions are combined by using algorithms such as Miner's rule, also known as Palmgren-Miner linear damage hypothesis. The algorithm assumes that there are m different stress magnitudes in a spectrum $\{S_i, i = 1..m\}$, each

contributing $n_i(S_i)$ cycles, and $N_i(S_i)$ is the number of cycles to failure of a constant stress S_i . The failure occurs when

$$\sum_{i=1}^m \frac{n_i(S_i)}{N_i(S_i)} = C_{\max}, \dots\dots\dots (1)$$

where C_{\max} is an experimental constant between 0.7 and 2.2. The rule (1) does not include handling of the probabilistic nature of fatigue. Also the effects of dynamic stress changes are not taken into account.

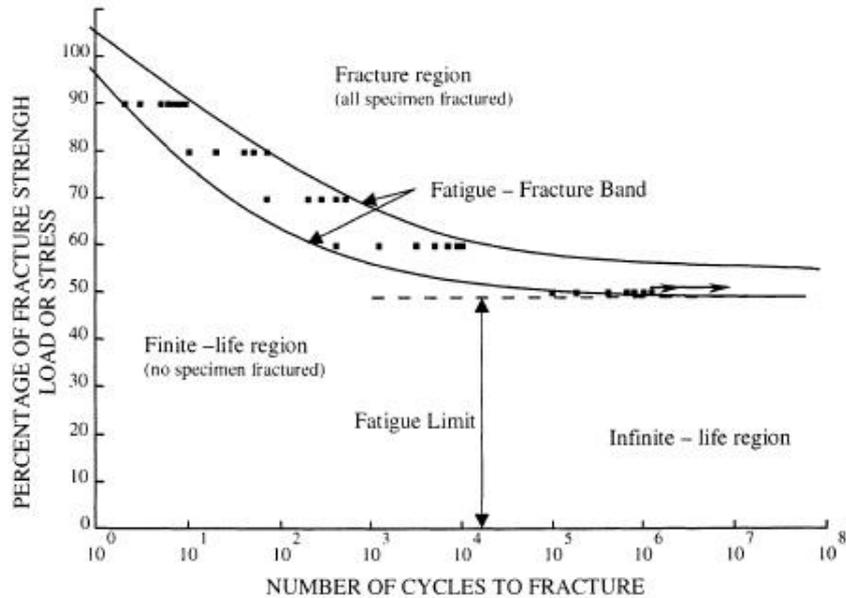


Figure 1. S–N curves in typifying fatigue test results (Bathias01, ASM86).

3. Stress indices

3.1 Signal processing and feature extraction

The generalised norm defined by

$$\left\| {}^\tau M_\alpha^p \right\|_p = \left({}^\tau M_\alpha^p \right)^{1/p} = \left(\frac{1}{N} \sum_{i=1}^N \left| x_i^{(\alpha)} \right|^p \right)^{1/p}, \dots\dots\dots (2)$$

where $\alpha \in \Re$ is the order of derivation, the order of the moment $p \in \Re$ is non-zero, τ is the sample time, has same dimensions as $x^{(\alpha)}$. The norm (2) include the norms from the minimum to the maximum, which correspond the orders $p = -\infty$ and $p = \infty$, respectively. The norm values increase with increasing order, i.e.

$$\left({}^\tau M_\alpha^p \right)^{1/p} \leq \left({}^\tau M_\alpha^q \right)^{1/q}, \dots\dots\dots (3)$$

if $p < q$. (CM08Signal,IJCM11a)

The normalised moments were in ^(CM10Ind) generalised by replacing $E(X)$ with the norm (2) as the central value c :

$$\gamma_k = \frac{E\left[\left(X^{(\alpha)} - \left\|{}^{\tau} M_{\alpha}^p\right\|_p\right)^k\right]}{\sigma_X^k}, \dots\dots\dots (4)$$

where σ_X is calculated about the origin, and k is a positive integer. The 3rd moment, skewness, is used in the analysis of the corner points, which define the scaling functions.

3.2 Scaling functions

Both expertise and data can be used in developing the mapping functions (membership definitions). The basic idea is to extract the meanings of variables from measurement signals. The scaling function scales the real values of variables to the range of [-2, +2] which combines normal operation [-1, +1] with the handling of warnings and alarms. The scaling function contains two monotonously increasing functions: one for the values between -2 and 0, and one for the values between 0 and 2. ^(Juuso04IIA)

The membership definition f consists of two second-order polynomials, i.e. the scaled values, which are called linguistic levels X_j , are obtained by means of the inverse function f^{-1} :

$$X_j = \begin{cases} 2 & \text{with } x_j \geq \max(x_j) \\ \frac{-b_j^+ + \sqrt{b_j^{+2} - 4a_j^+(c_j - x_j)}}{2a_j^+} - 2 & \text{with } c_j \leq x_j \leq \max(x_j) \dots\dots\dots (5) \\ \frac{-b_j^- + \sqrt{b_j^{-2} - 4a_j^-(c_j - x_j)}}{2a_j^-} - 2 & \text{with } \min(x_j) \leq x_j \leq c_j \\ -2 & \text{with } x_j \leq \min(x_j) \end{cases}$$

where a_j^- , b_j^- , a_j^+ and b_j^+ are coefficients of the corresponding polynomials, c_j is a real value corresponding to the linguistic value 0 and x_j is the actual measured value. Parameters $\min(x_j)$ and $\max(x_j)$ corresponding to the linguistic values -2 and 2 define the support. The coefficients of the polynomials are defined by points

$$\{(\min(x_j), -2), ((c_l)_j, -1), (c_j, 0), ((c_h)_j, 1), (\max(x_j), 2)\}, \dots\dots\dots (6)$$

where $[(c_l)_j, (c_h)_j]$ is the core.

The centre point, c_j , defines the operating point. Four linear equations are needed for solving the other coefficients:

$$\begin{aligned} 4a_j^- - 2b_j^- + c_j &= \min(x_j) \\ a_j^- - b_j^- + c_j &= (c_l)_j \\ a_j^+ + b_j^+ + c_j &= (c_h)_j \\ 4a_j^+ + 2b_j^+ + c_j &= \max(x_j) \end{aligned} \dots\dots\dots (7)$$

The scaling functions are monotonously increasing if the coefficients,

$$\alpha_j^- = \frac{(c_l)_j - \min(x_j)}{c_j - (c_l)_j} = \frac{(c_l)_j - \min(x_j)}{\Delta c_j^-}, \dots \dots \dots (8)$$

$$\alpha_j^+ = \frac{\max(x_j) - (c_h)_j}{(c_h)_j - c_j} = \frac{\max(x_j) - (c_h)_j}{\Delta c_j^+}, \dots \dots \dots$$

are both in the range $\left[\frac{1}{3}, 3\right]$. Corrections are done by changing the borders of the core area, the borders of the support area or the centre point. Additional constraints for derivatives can also be taken into account. The coefficients of the polynomials can be represented by

$$a_j^- = \frac{1}{2}(1 - \alpha_j^-) \Delta c_j^-,$$

$$b_j^- = \frac{1}{2}(3 - \alpha_j^-) \Delta c_j^-, \dots \dots \dots (9)$$

$$a_j^+ = \frac{1}{2}(\alpha_j^+ - 1) \Delta c_j^+,$$

$$b_j^+ = \frac{1}{2}(3 - \alpha_j^+) \Delta c_j^+,$$

where $\Delta c_j^- = c_j - (c_l)_j$ and $\Delta c_j^+ = (c_h)_j - c_j$. Membership definitions may contain linear parts if some coefficients α_j^- or α_j^+ equals to one.

The best way to tune the system is to first define the working point and the core, then the ratios α_j^- and α_j^+ from the range $\left[\frac{1}{3}, 3\right]$, and finally to calculate the support. The membership definitions of each variable are configured with five parameters, including the centre point c_j and three consistent sets:

- corner points $\{ \min(x_j), (c_l)_j, (c_h)_j, \max(x_j) \}$ are good for visualisation,
- parameters $\{ \alpha_j^-, \Delta c_j^-, \alpha_j^+, \Delta c_j^+ \}$ are suitable for tuning, and
- coefficients $\{ a_j^-, b_j^-, a_j^+, b_j^+ \}$ are used in the calculations.

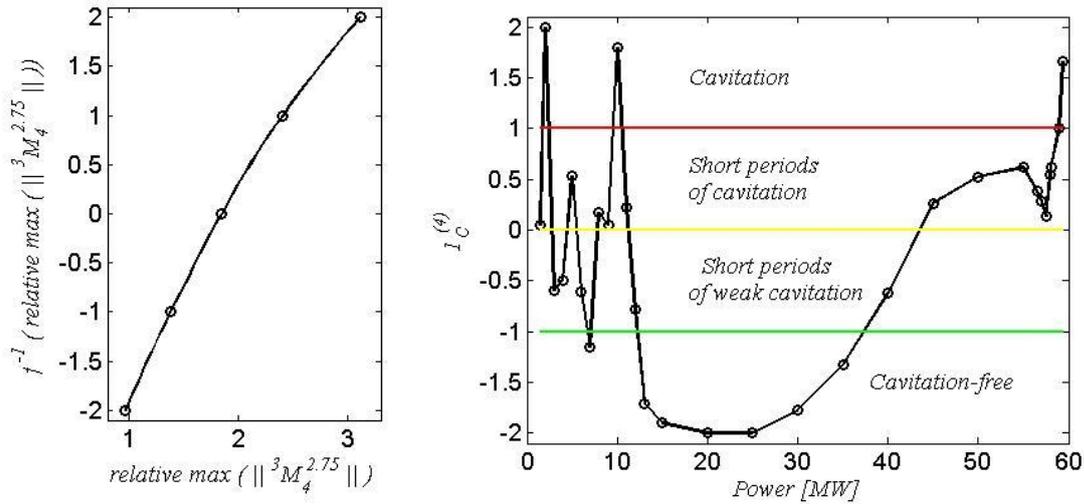
The upper and the lower parts of the scaling functions can be convex or concave, independent of each other. Simplified functions can also be used, e.g. a linear membership definition requires two and an asymmetrical linear definition three parameters. Additional constraints can be taken into account for derivatives, e.g. locally linear function results if continuous derivative is chosen in the centre point. (ICANNGA09)

Additional constraints can be taken into account for derivatives, e.g. a good solution can be to use a locally linear function in the neighbourhood of the centre point. The continuous derivative is achieved if

$$6c_j - 4(c_l)_j - 4(c_h)_j + \min(x_j) + \max(x_j) = 0. \dots \dots \dots (10)$$

This can be achieved by modifying the centre point or the corner points of the feasible range. There are several acceptable modification alternatives.

The analysis of the corner points has earlier based on mean or median values. The value range of x_j is divided into two parts by the central tendency value c_j and the core area, $[(c_l)_j, (c_h)_j]$, is limited by the central tendency values of the lower and upper part. There are problems when the value range is very wide or the distribution is very concentrated. The approach based on (4) was introduced in ^(CM10Ind) for estimating the central tendency value and the core area, $\alpha = 0$. The central tendency value is chosen by the point where the skewness changes from negative to positive, i.e. $\gamma_3 = 0$. Then the data set is divided into two parts: a lower part and an upper part. The same analysis is done for these two data sets. The estimates of the corner points, $(c_l)_j$ and $(c_h)_j$, are the points where the direction of the skewness changes. The iteration is performed with generalised norms. Then the ratios α_j^- and α_j^+ are restricted to the range $\left[\frac{1}{3}, 3\right]$ moving the corner points $(c_l)_j$ and $(c_h)_j$ or the upper and lower limits $\min(x_j)$ and/or $\max(x_j)$. The linearity requirement (10) is taken into account, if possible. ^(CM10Ind)



(a) Scaling function.

(b) Cavitation index.

Figure 2. Scaling function of the relative $\max(\|{}^3M_4^{2.75}\|)$ and the corresponding cavitation index $I_C^{(4)}$.

3.3 Condition and stress indices

Cavitation indices obtained from the scaled values ^(CM10Ind) provide an indication of the severity of the cavitation. The index shown in Figure 2 is calculated by using the generalised norms $\|{}^3M_4^{2.75}\|$ and the nonlinear scaling based on skewness. The indices are calculated with problem-specific sample times, and variation with time is handled as uncertainty by presenting the indices as time-varying fuzzy numbers. The classification limits can also be considered fuzzy. Practical long-term tests have been performed e.g.

for diagnosing faults in bearings, in supporting rolls of lime kilns and for the cavitation of water turbines ^(CM10Ind). The indices obtained from short samples are aimed for use in the same way as the process measurements in process control. The new indices are consistent with the measurement and health indices developed for condition monitoring. ^(CM08Ind) The cavitation index is an example of a stress index: $I_s = -2$ when the stress is negligible, and levels $\{-1, 0, 1\}$ are analogue to the lower limits of the vibration severity ranges {usable, still acceptable, not acceptable} defined in the VDI 2056 ^(VDI64,Collacott77).

3.4 LE Models

The LE models are linear equations

$$\sum_{j=1}^m A_{ij} X_j + B_i = 0, \dots\dots\dots (11)$$

where X_j is a linguistic level for the variable $j, j=1..m$. Each equation i has its own set of interaction coefficients $A_{ij}, j=1..m$. The bias term B_i was introduced for fault diagnosis systems. Various fuzzy models can be represented by means of LE models, and neural networks and evolutionary computing can be used in tuning. The first LE application in condition monitoring was presented in ^(Eunite04). The condition monitoring applications are similar to the applications intended for detecting operating conditions in the process industry ^(PohTO05).

4. Fatigue prediction

Wöhler curves are represented by a linguistic equation

$$I_s = \log_{10}(N_c), \dots\dots\dots (12)$$

where the stress index can be a scaled value of stress, $f^{-1}(S_i)$, or a scaled value of a generalised norm obtained from vibration signals: $f^{-1}(\|M_\alpha^p\|)$. The scaling of the logarithmic values of the number of cycles, N_c , is linear. As the LE model is nonlinear, it covers a wide operating range. The system may also contain several specific equations corresponding different operating point, e.g. low, normal and high stress.

The Wöhler curves can be generated from material tests. For existing Wöhler curves, the scaling functions of the stress are generated by defining the corner points (6) from the selected points (S, N_c) . Then corner points are modified if the limits of the shape factors α_j^- and α_j^+ are violated. Locally linear function in the neighbourhood of the centre point can be introduced if (10) is feasible. The set of equations (7) can be used for $S-N$ test results.

For process equipments, the $S-N$ curves are gradually refined since extensive tests cannot be performed in the same way as for materials. The approach is similar to the one used in recursive modelling for prognostics ^(CM11Recur).

The continuous model (12) extends the principle of the Palmgren-Miner linear damage hypothesis (1). In each sample time, τ , the cycles $N_c(k)$ obtained from $I_s(k)$ by (12), and the resulting contribution $\frac{\tau}{N_c(k)}$ summarised to the previous contributions

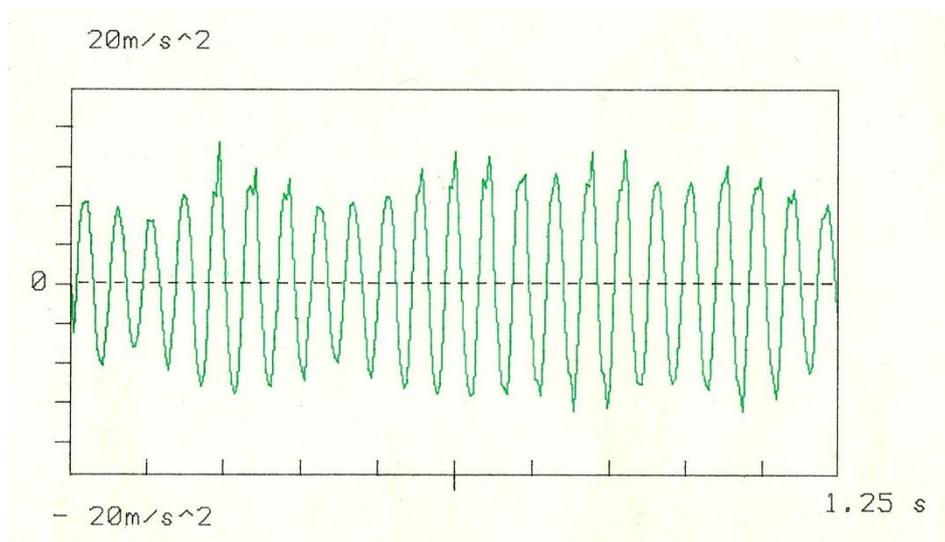
$$C(k) = C(k-1) + \frac{\tau}{N_c(k)}, \dots\dots\dots (13)$$

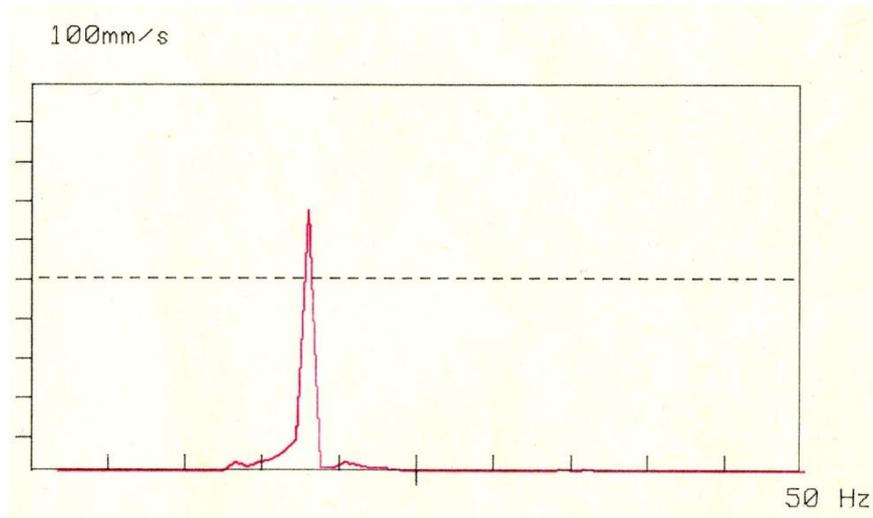
which can also be used for predictions based on scenarios of the use. Since the stress is not constant for the whole cycle, the sample time is taken as a fraction of the cycle time. The previous history can be updated whenever the scaling functions are changed.

The cumulative sum of the contributions presented by (13) indicates the deterioration of the condition, and the simulated sums can be used for predicting the failure time. The high stress contributions dominate in the summation. Correspondingly, the very low stress periods have a negligible effect, which is consistent with the idea of infinite life time (Figure 1). The summation of the contributions also reveals repeated loading and unloading, and the individual contributions provide indications of the severity of the effect.

5. Applications

Feature extraction is based on velocity $x^{(1)}$, acceleration $x^{(2)}$ and higher derivatives, $x^{(3)}$ and $x^{(4)}$. The other signals have been obtained from acceleration through analogue ⁽¹¹⁾ or numerical integration and derivation ⁽¹⁵⁾.





6. Conclusions

The Wöhler curves represented by linguistic equation (LE) models are feasible in calculating contributions of complex load, which is varying with time. Also random load can be handled with simulation. The previous history can be updated whenever the scaling functions are changed. The cumulative sum of the contributions indicates the deterioration of the condition, and the simulated sums can be used for predicting the failure time. The summation of the contributions also reveals repeated loading and unloading, and the individual contributions provide indications of the severity of the effect. Effects of dynamic stress changes can be taken into account by including stress changes in the calculation of the contributions during the appropriate sample times.

References

1. (Insight07)S Lahdelma and E Juuso, 'Advanced signal processing and fault diagnosis in condition monitoring', *Insight*, Vol 49, No 12, pp 719–725, 2007.
2. (CM08Signal)S Lahdelma and E Juuso, 'Signal Processing in Vibration Analysis', *Proceedings CM 2008 – MFPT 2008, 5th International Conference on Condition Monitoring & Machinery Failure Prevention Technologies*, Edinburgh, Coxmoor, Oxford, UK, pp 867-878, July 2008.
3. (Safeprocess94)E K Juuso, 'Fault Diagnosis based on Linguistic Equation Framework', *Preprints of SAFEPROCESS'94*, Espoo, Finland, pp 374-379, Hakapaino, Helsinki, Finland, June 1994.
4. (Juuso99)E K Juuso, 'Control in Process Industry: The Linguistic Equation Approach', in H B Verbruggen, H-J Zimmermann and R Babuska (Eds.), *Fuzzy Algorithms for Control, International Series in Intelligent Technologies*, 1999, Kluwer, Boston, pp 243-300.
5. (Juuso04IJA)E K Juuso, 'Integration of Intelligent Systems in Development of Smart Adaptive Systems', *International Journal of Approximate Reasoning*, Vol 35, No 3, pp 307-337, 2004.

6. (IFAC92)E K Juuso and K Leiviskä, 'Adaptive expert systems for metallurgical processes, in expert systems in mineral and metal processing', IFAC Workshop Series, No 2, Pergamon, Oxford, UK, pp 119-124, 1992.
7. (Eunite04)E K Juuso, M Kivistö and S Lahdelma, 'Intelligent Condition Monitoring Using Vibration Signals', Proceedings of EUNITE 2004, Aachen, Germany, Verlag Mainz, Aachen, pp 381-390, June 2004.
8. (PohTO05)E Juuso and K Leiviskä, 'Combining Monitoring and Process Data in Detecting Operation Conditions in Process Industry', Maintenance, Condition Monitoring and Diagnostics, Proceedings of the 2nd International Seminar, Oulu, Finland, pp 145-156, September 2005.
9. (CM08Ind)E Juuso and S Lahdelma, 'Intelligent Condition Indices in Fault Diagnosis', Proceedings CM 2008 – MFPT 2008, 5th International Conference on Condition Monitoring & Machinery Failure Prevention Technologies, Edinburgh, Coxmoor, Oxford, UK, pp 698-708, July 2008.
10. S Lahdelma and E Juuso, 'Signal Processing and Feature Extraction in Vibration Analysis - Part I: Methodology', submitted to Journal of Condition Monitoring.
11. E Juuso and S Lahdelma, 'Intelligent Cavitation Indicator for Kaplan Water Turbines', Proceedings of COMADEM 2006, Luleå, Sweden, 2006, pp 849-858, Luleå University Press, June 2006.
12. S Lahdelma, E Juuso and J Strackeljan, 'Neue Entwicklungen auf dem Gebiet der Wälzlagerüberwachung', Tagungsband zum AKIDA 2006, Aachen, Germany, pp 447-460, ASRE, Band 63, 2006, RWTH Aachen, November 2006.
13. S Lahdelma and E K Juuso, 'Intelligent condition monitoring for lime kilns', Tagungsband zum AKIDA 2006, Aachen, Germany, pp 399-408, ASRE, Band 63, RWTH Aachen, November 2006.
14. (ICANNGA09)E K Juuso, 'Tuning of large-scale linguistic equation (LE) models with genetic algorithms', Adaptive and Natural Computing Algorithms, Revised selected papers - ICANNGA 2009, Kuopio, Finland ICANNGA 2009, Lecture Notes in Computer Science (LNCS) 5495, pp 161-170, Springer, Heidelberg, 2009.
15. S Lahdelma, P Vähöja, and E Juuso, 'Detection of cavitation in Kaplan water turbines', Proceedings of 2007 Arctic Summer Conference on Dynamics, Vibrations and Control, Ivalo, Finland, pp 80-87, Tampere University Press, Tampere, Finland, August 2007.
16. E Juuso and S Lahdelma, 'Cavitation indices in power control of Kaplan water turbine', Proceedings CM 2009 – MFPT 2009, 6th International Conference on Condition Monitoring & Machinery Failure Prevention Technologies, Dublin, Ireland, Coxmoor, Oxford, UK, pp 831-842, June 2009.
17. (VD64)VDI 2056 Beurteilungsmaßstäbe für mechanische Schwingungen von Maschinen, VDI-Richtlinien, Oktober 1964.
18. (Collacott77)R A Collacott, 'Mechanical Fault Diagnosis and condition monitoring', Chapman and Hall, London, 1977.
19. S Lahdelma and E Juuso, 'Generalised lp Norms in Vibration Analysis of Process Equipments', Proceedings CM 2010 – MFPT 2010, 7th International Conference on Condition Monitoring & Machinery Failure Prevention Technologies, Stratford-upon-Avon, UK, Coxmoor, Oxford, UK, June 2010.
20. (IJCM11a)S Lahdelma and E Juuso, 'Signal Processing and Feature Extraction in Vibration Analysis, Part I: Methodology', The International Journal of Condition Monitoring, Vol 1, No 2, pp 46-53, 2011.

21. (IJCM11b)S Lahdelma and E Juuso, 'Signal Processing and Feature Extraction in Vibration Analysis, Part II: Applications', *The International Journal of Condition Monitoring*, Vol 1, No 2, pp 54-66, 2011.
22. (Bathias99)C Bathias, 'There is no infinite fatigue life in metallic materials', *Fatigue & Fracture of Engineering Materials & Structures*, Vol 22, No 7, pp 559-565, 1999.
23. (ASM86)Atlas of Fatigue Curves, H E Boyer (ed.), ASM International, 518 pp, 1986.
24. (StephensFuchs01)R I Stephens and H O Fuchs, *Metal Fatigue in Engineering*, 2nd edition, John Wiley & Sons, 2001.
25. (Schutz96)W Schütz, 'A history of fatigue', *Engineering Fracture Mechanics*, Vol 54, No 2, pp 263-300, 1996.
26. (MarinesBB03)I Marines, X Bin and C Bathias, 'An understanding of very high cycle fatigue of metals', *International Journal of Fatigue*, Vol 25, pp 1101–1107, 2003.
27. (CM10Ind)E Juuso and S Lahdelma, 'Intelligent scaling of features in fault diagnosis', *Proceedings The 7th International Conference on Condition Monitoring and Machinery Failure Prevention Technologies*, Stratford-upon-Avon, UK. BINDT, 15 pp, June 2010.
28. (CM11Recur) E Juuso and S Lahdelma, 'Intelligent Trend Indices and Recursive Modelling in Prognostics', *Proceedings CM 2011 – MFPT 2011, 8th International Conference on Condition Monitoring and Machinery Failure Prevention Technologies*, Cardiff, UK. BINDT, 17 pp, June 2011.