Stress Indices in Fatigue Prediction

Esko K. Juuso Mika Ruusunen

Control Engineering Group, Faculty of Technology, P.O.Box 4300, 90014 University of Oulu, Finland
(e-mail: esko.juuso@oulu.fi, mika.ruusunen@oulu.fi)

Abstract: Fatigue is caused by cracks formed in repeated loading and unloading situations when the load exceeds certain thresholds. Structures fracture suddenly when a crack reaches a critical size. Intelligent stress indices based on nonlinear scaling provide good indicators of the severity of the load. The stress index is -2 when the stress is negligible, and levels \{-1, 0, 1\} are analogue to the lower limits of the vibration severity ranges usable, still acceptable, not acceptable. The Wöhler curve represented by a linguistic equation (LE) model makes the calculations highly efficient. The contribution of the stress is calculated in each sample time, which corresponds the cycle time. The cumulative sum of the contributions indicates the deterioration of condition and provides a prediction of the fatigue risk. In this case, torque measurements collected from a roller mill have been analysed by using a combination of two norms and the stress index is obtained by nonlinear scaling from the combination of these features. Long operating periods can be achieved if the stress levels are kept low, in practice a large number of passes have low stress indices. The high stress situations are seen as a very steep rise. The stress levels can be followed by a generalised statistical process control approach. At the risk level higher than 60%, a single high torque level can have a strong effect on the activation of a failure. These approaches operate well for the set of failures analysed in this study and is promising for practical use.

Keywords: Fatigue detection, nonlinear scaling, intelligent stress indices, torque measurements, condition monitoring

1. INTRODUCTION

Rolling mills are heavily loaded process equipment, which require condition monitoring solutions to maximise utilisation at a reduced risk of failures. Torque monitoring enables a condition-based maintenance of main drive spindles to optimise rolling schedules and minimise the risk of overloads. Torque sensor technology presented is specifically designed for rough ambient conditions. When loads are increased monitoring of critical components becomes more and more important. Overload risk rises and service factor of equipment is reduced. The key point is to maintain or upgrade equipment as late as possible, but as early as necessary. The higher the capital asset of the equipment and the more disastrous the potential consequences of a failure are, the higher the necessity for condition monitoring. Typical areas of application are drive spindles, gear boxes, roll housings and electrical motors. (Mackel and Fieweger, 2010)

The history of fatigue analysis already began in 1837, when Wilhelm Albert published the first fatigue test results (Schütz, 1996) in Clausthal. Wöhler concluded that cyclic stress range is more important than peak stress and introduces the concept of the endurance limit. Fatigue is progressive, localised structural damage that is caused by repeated loading and unloading. The nominal maximum stress values are less than the ultimate tensile stress limit and may be below the yield stress limit of the material. The mechanism proceeds through cracks formed when the load exceeds certain thresholds. Structures fracture suddenly when a crack reaches a critical size. The shape of the structure will significantly affect fatigue life; square holes or sharp corners will lead to elevated local stresses where fatigue cracks can initiate. Round holes and smooth transitions or fillets are therefore important in order to increase the fatigue strength of the structure. The effects of each stress level are taken into account in the calculations of cumulative damage from individual contributions (Palmgren, 1924; Miner, 1945).

Advanced signal processing methods and intelligent fault diagnosis have been developed to detect different types of machine faults reliably at an early stage. Dimensionless indices, which are obtained by comparing each feature value with the corresponding value in normal operation, provide useful information on different faults, and even more sensitive solutions can be obtained by selecting suitable features. (Lahdelma and Juuso, 2007) Generalised moments and norms include many well-known statistical features as special cases and provide compact new features capable of detecting faulty situations (Lahdelma and Juuso, 2008, 2011a,b).

Intelligent methods extend the idea of dimensionless indices to nonlinear systems: the basic idea is nonlinear scaling, which was developed to extract the meanings of variables from measurement signals (Juuso, 2004). In the present systems, the scaling functions are developed by using generalised moments and norms (Juuso and Lahdelma, 2010; Juuso, 2013) and tuned with genetic algorithms (Juuso, 2009).
The condition monitoring applications are similar with detecting operating conditions in the process industry (Juuso and Leiviskä, 2010). Detection of operating conditions can be extended by means of a Case-Based Reasoning (CBR) type application with linguistic equation (LE) models and fuzzy logic (Juuso, 1994, 1999, 2004). The basic idea is nonlinear scaling, which was developed to extract the meanings of variables from measurement signals (Juuso and Leiviskä, 1992). The parameters of the scaling functions can be recursive updated with data analysis: the scaling is upgraded gradually and even the initial estimates are not necessary (Juuso, 2015b).

Torque measurement technology has been discussed in (Mackel and Fieweger, 2010). Intelligent stress indices based on nonlinear scaling were introduced to fatigue detection in (Juuso and Lahdelma, 2012). The Wöhler curve is represented by a linguistic equation (LE) model, where the stress index can be a scaled value of stress or a scaled value of a generalised norm obtained from vibration signals. Torque measurements are informative in fatigue prediction (Juuso and Ruusunen, 2013). A generalised statistical process control (GSPC) for stress monitoring by using the nonlinear scaling methodology to evaluate limits (Juuso, 2015a).

This paper addresses fatigue prediction using intelligent stress indices obtained from torque measurements in a roller mill. Advanced signal processing, generalised norms and nonlinear scaling are combined in the calculations.

2. TORQUE MONITORING

Torque is one of the most important rolling process measurements since the whole power required for material forming is transmitted via drive trains to the rolls. In practice the main drives of the rolling mill are highly dynamically loaded, e.g. bite impact, reversing rolling practice, torsional vibrations and cobbling affect the product and the residual life time of drive components. The torque sensors used in rolling mills have to be extremely robust because of the rough operating conditions. Although often neglected because of cost reasons, torque monitoring provides efficient tools for product quality and plant reliability. The monitoring of rolling mill main drives requires torque to be measured directly at spindles or motor shafts. The current signal from the motor, which is often misused for similar monitoring purposes offers only limited information, especially in terms of signal dynamics (Figure 1). Measured rolling torque will thus become an important measurement for general process monitoring and an essential information for drive train monitoring. (Mackel and Fieweger, 2010)

3. STRESS INDICES

3.1 Nonlinear scaling

Meanings of feature and index levels are essential in stress monitoring. Membership functions used in fuzzy logic are represented with membership definitions, which provide nonlinear mappings from the operation area, defined with feasible ranges, to the linguistic values represented inside a real-valued interval [-2,2]. The basic scaling approach presented in (Juuso, 2004) has been improved later: a new constraint handling was introduced in (Juuso, 2009), and a new skewness based methodology was presented for signal processing in (Juuso and Lahdelma, 2010). Membership definitions are monotonously increasing scaling functions \( f() \) which consist of two second order polynomials and their inverse functions \( f^{-1}() \).

3.2 Feature extraction

Features are extracted from measurements with generalised norms defined by

\[
||\tau M^p||_p = (\tau M^p)^{1/p} = \left(\frac{1}{N} \sum_{i=1}^{N} x_j^p\right)^{1/p},
\]

where \( p \neq 0 \), is calculated from \( N \) values of a sample, \( \tau \) is the sample time. With a real-valued order \( p \in \mathbb{R} \) this norm can be used as a central tendency value if \( ||\tau M^p||_p \in \mathbb{R} \), i.e. \( x_j > 0 \) when \( p < 0 \), and \( x_j \geq 0 \) when \( p > 0 \). The norm (1) is calculated about the origin, and it combines two trends: a strong increase caused by the power \( p \) and a decrease with the power \( 1/p \). Therefore, all the norms have same dimensions as \( x_j \). The generalised norm of absolute values \( |x_j| \) was introduced for signal analysis in (Lahdelma and Juuso, 2008). In stress monitoring, all the features and indices are positive.

3.3 Scaling functions

Monotonously increasing scaling functions can be constructed by adjusting the centre point \( c_j \), the core \([c_j, c_j] \) and the support \([\min(x_j), \max(x_j)] \). In the data-based solution, the value range of \( x_j \) is divided into two parts by the central tendency value \( c_j \) and the core area, \([c_j, c_j] \), is limited by the central tendency values of the lower and upper part. The approach is based on the normalised moments generalised by replacing the expectation with the norm (1) as the central value:

\[
\gamma_k = \frac{1}{N \sigma_j^k} \sum_{i=1}^{N} (x_j - ||\tau M^p||_p)^k
\]

where \( \sigma_j \) is calculated about the origin, and \( k \) is a positive integer. (Juuso and Lahdelma, 2010)
The monotonous increase is achieved with a sequential approach introduced in (Juuso, 2000): first define the centre point \( c_j \), then the core by choosing the ratios
\[
\alpha_j^+ = \frac{c_j - \min(x_j)}{c_j - (c_j)_1}, \quad \alpha_j^- = \frac{c_j - \max(x_j)}{(c_j)_m - c_j}
\]
from the range \([-0.5, 0.5]\), and finally calculate the support \([\min(x_j), \max(x_j)]\). The norms (1) are used together with the generalised skewness (2) in the data-driven approach to define the centre and corner points. The ratios (3), which are checked in all data-driven cases, are also guiding the manual construction of the scaling functions. Additional constraints are used e.g. to introduce local linear parts can be included if they are feasible.

The nonlinear scaling methodology provides good results for the automatic generation of scaling functions. Even small faults and anomalies are detected. The approach has been tested with normal, Poisson and Weibull distributions and used in condition monitoring applications (Juuso and Lahdelma, 2010). This approach is suitable for a very large set of statistical distributions (Juuso, 2013).

### 3.4 Stress indices

Stress indices obtained from the scaled values (Juuso and Lahdelma, 2010) provide an indication of the severity of the load. The indices are calculated with problem-specific sample times, and variation with time is handled as uncertainty by presenting the indices as time-varying fuzzy numbers. The classification limits can also be considered fuzzy. Practical long-term tests have been performed e.g. for diagnosing faults in bearings, in supporting rolls of lime kilns and for the cavitation of water turbines (Juuso and Lahdelma, 2010). The indices obtained from short samples are aimed for use in the same way as the process measurements in process control. The new indices are consistent with the measurement and health indices developed for condition monitoring. (Juuso and Lahdelma, 2008) The cavitation index is an example of a stress index: when the stress in negligible, and levels -1, 0, 1 are analogue to the lower limits of the vibration severity ranges usable, still acceptable, not acceptable defined in the VDI 2056 (VDI, 1964; Collacott, 1977).

### 3.5 LE models

The nonlinear scaling transforms the nonlinear model \( y = F(x) \) to a linear problem. The basic element of a linguistic equation (LE) model is a compact equation
\[
\sum_{j=1}^{m} A_{ij} X_j(t - n_j) + B_i = 0, \tag{4}
\]
where \( X_j \) is a linguistic value for the variable \( j, j = 1...m \). Each variable \( j \) has its own time delay \( n_j \) compared to the variable with latest time label. Linguistic values in the range \([-2, 2]\) are obtained from the actual data values by membership definitions. The directions of the interaction are represented by interaction coefficients \( A_{ij} \in \mathbb{R} \). In the original system (Juuso and Leiviskä, 1992), the linguistic labels \{very low, low, normal, high, very high\} were replaced by numbers \{-2, -1, 0, 1, 2\}.

The coefficients \( A_{ij} \) and \( B_i \) in (4) have a relative meaning, i.e. the equation can be multiplied or divided by any non-zero real number without changing the model. A LE model with several equations can be represented as a matrix equation
\[
AX + B = 0, \tag{5}
\]
where the interaction matrix \( A \) contains all coefficients \( A_{ij}, i = 1, \ldots, n, j = 1, \ldots, m \), and the bias vector \( B \) all bias terms \( B_i, i = 1, \ldots, n \). The time delays of individual variables are equation specific. As linear equations, each model can be used in any direction, i.e. the output variable can be chosen freely.

![Fig. 2. S-N curves in typifying fatigue test results (Bathia, 1999; Boyer, 2001).](image)

**4. FATIQUE**

### 4.1 Stress and fatigue

ASTM International, earlier known as the American Society for Testing and Materials, defines fatigue life, \( N_f \), as the number of stress cycles of a specified character that a specimen sustains before failure of aspecific nature occurs. (Stephens and Fuchs, 2001) In high-cycle fatigue situations, material performance is commonly characterized by an S-N curve, also known as the Wöhler curve (Figure 2). This is a graph of the magnitude of a cyclic stress \( S \) against the logarithmic scale of cycles to failure \( N \). The curves are material specific (Marines et al., 2003). S-N curves are derived from tests on samples of the material where a regular sinusoidal stress is applied by a testing machine which also counts the number of cycles to failure. Probability distributions that are common in data analysis and in design against fatigue include the lognormal distribution, extreme value distribution, the Birnbaum-Saunders distribution and the Weibull distribution. (Juuso and Lahdelma, 2012)

In practice, the sequence of load is complex, often random, including large and small loads. The rainflow analysis and histograms of cyclic stress are used to assess the safe life in these cases. The effects of each stress level are taken into account in the calculations of cumulative damage. Individual contributions are combined by means of algorithms such as the Miners rule, also known as the
Palmgren-Miner linear damage hypothesis. The algorithm assumes that there are \( m \) different stress magnitudes in a spectrum \( \{ S_i, i = 1 \ldots m \} \), each contributing \( n_i(S_i) \) cycles, and \( N_i(S_i) \) is the number of cycles to the failure of a constant stress \( S_i \). The failure occurs when

\[
\sum_{i=1}^{m} \frac{n_i(S_i)}{N_i(S_i)} = C_{\text{max}}
\]  

where \( C_{\text{max}} \) is an experimental constant between 0.7 and 2.2. The rule (6) does not include the handling of the probabilistic nature of fatigue. The effects of dynamic stress changes are not taken into account either. (Juuso and Ruusunen, 2013)

Fatigue and wear monitoring for the condition-based maintenance of torque loaded drive equipment can be improved by compiling load collectives and condensing these load collectives into box-plots for each fatigue affected drive component (Figure 3).

4.2 Fatigue prediction

Wöhler curves were in (Juuso and Lahdelma, 2012) represented by a linguistic equation

\[
I_S = \log_{10}(N_C)
\]  

where the stress index can be a scaled value of stress, \( f^{-1}(S_i) \), or a scaled value of a generalised norm obtained from vibration signals: \( f^{-1}(||\tau M_p\||) \). The scaling of the logarithmic values of the number of cycles, \( N_C \), is linear. As the LE model is nonlinear, it covers a wide operating range. The system may also contain several specific equations corresponding to different operating points, e.g. low, normal and high stress.

The Wöhler curves can be generated from material tests. For the existing Wöhler curves, the scaling functions of the stress are generated by defining the corner points from the selected points \( (S, N_C) \). Then the corner points are modified if the limits of the shape factors \( \alpha_j - \alpha_j^+ \) are violated. For process equipment, the S-N curves are gradually refined, as extensive tests cannot be performed in the same way as for materials. The approach is similar to the one used in recursive modelling for prognostics (Juuso and Lahdelma, 2011; Juuso, 2015b).

The continuous model (7) extends the principle of the Palmgren-Miner linear damage hypothesis (6). In each sample time, \( \tau \), the cycles \( N_C(k) \) obtained from \( I_S(k) \) by (7), and the resulting contribution \( \frac{\tau}{N_C(k)} \) summarised to the previous contributions

\[
C(k) = C(k-1) + \frac{\tau}{N_C(k)}
\]  

which can also be used for predictions based on use scenarios. Since the stress is not constant for the whole cycle, the sample time is taken as a fraction of the cycle time. The previous history can be updated whenever the

![Fig. 3. Load-classification box-plots show fatigue and utilisation ratio for equipment such as spindles or rolls (Mackel and Fieweger, 2010).](image-url)
scaling functions are changed. (Juuso and Lahdelma, 2012) The value range of the sum $C$ is scaled to provide the fatigue risk in percents (%).

The cumulative sum of the contributions presented by (8) indicates the deterioration of the condition, and the simulated sums can be used to predict the failure time. The high stress contributions dominate in the summation. Correspondingly, the very low stress periods have a negligible effect, which is consistent with the idea of infinite life time (Figure 2). The summation of the contributions also reveals repeated loading and unloading, and the individual contributions provide indications of the severity of the effect.

5. ROLLER MILL

The condition monitoring approach based on stress indices has been tested by using torque measurements from a hot rolling mill. The aims are to calculate a prediction of the fatigue risk and to provide information for expanding the feasible operating time.

5.1 Measurements

Data preparation from the huge measurement material was a very time-consuming phase in this project: torque measurements were analysed from 35000 passes, each having measurement values in the similar way as shown in Figure 1. The maximum torques have important effect on the progress of the fatigue risk, e.g. very high torques can cause an immediate failure. Three very high torque values are seen in Figure 4.

5.2 Features

The feature alternatives included several norms and their combinations. The feature selection is based on the information theory: the best similarity between signals is achieved for a feature obtained as difference between the effective and average values, i.e.

$$
Feature = \frac{1}{N} \sum_{i=1}^{N} (x_j)_{i}^{2} - \frac{1}{N} \sum_{i=1}^{N} (x_j)_{i}
$$

(9)

where $x_j$ is the fillet split. The time interval $\tau$ can be different for the passes. Since the orders of the norm are here 1 and 2, also negative values of $x_j$ can be used. The resulting features are shown in Figure 5.

5.3 Stress index

The scaling function is highly nonlinear (Figure 6). The support area is changed to ensure that the scaling function is monotonously increasing. All the feature values are positive: the negative value corresponding to the very low level is needed for function definitions. The shape is consistent with S-N curves shown in Figure 2.

5.4 LE based S-N curve

The S-N curve is represented by the linguistic equation (7). The stress index is calculated from the scaled features obtained by using the nonlinear scaling functions, also denoted as membership definitions. The resulting linguistic S-N curve is linear, which is clearly seen in Figure 7. The scaling function shown in Figure 6 extracts well the nonlinear effects. A large number of passes have low stress indices. The index values are scaled to the feature values to form a normal S-N curve (Figure 8). The high stress cases are seen as a very steep rise in the semilogarithmic curve which supports the previous idea to count only the number of overloads, see Figure 3.

5.5 Fatigue risk

The fatigue risk is predicted with the extended approach introduced in (Juuso and Lahdelma, 2012): the contribution of each pass is obtained by the continuous model (7) and summarised to the previous contributions by (8). The
risk is increasing fast when the stress index is high, but the increase can also be very slow for a quite long time (Figure 9) if the stress is kept in moderate levels.

At the risk level higher than 60%, a single high torque level can have a strong effect on the activation of a failure. The first failure at the pass 6181 takes place when the torque is very high, over 1 kNm (Figure 5). High torque values at the passes 24582 and 33606 do not cause a failure. There is a long operation period before the third failure at the pass 22164. The second failure does not fit this model.

The approach is promising for practical use since it operates well for the limited set of failures analysed in this study. However, further tuning and testing with the huge measurement material is needed.

The fatigue risk should be calculated separately for each workroll resulting a classification plot which refines the idea shown in Figure 3. The number of overloads is replaced by a quantitative, tunable fatigue risk calculated by (8).

5.6 Statistical process control

Statistical process control (SPC) is a feasible solution for demonstrating real time the risky levels of the stress during the operation, see Figure 10. SPC is based on continuously analysing and reducing variation in manufacturing processes (Oakland, 2008). Focus is on early detection and various control charts have been developed and widely used for that: Shewhart started already in 1920s. Standard control charts are often based on normal distributions, but non-Gaussian data need to be analysed in many cases. A flexible family of statistical distributions is applied in (Fournier et al., 2006).

In the applications, which require stress monitoring, both process measurements and condition monitoring measurements are highly nonlinear. The statistical process control (SPC) can be extended to nonlinear and non-Gaussian data by using the new generalised SPC introduced in
Juuso, E. and Lahdelma, S. (2010). Intelligent scaling of
Fournier, B., Rupin, N., Bigerelle, M., Najjar, D., and Iost,
Juuso, E.K. (2009). Tuning of large-scale linguistic
Juuso, E.K. (2004). Integration of intelligent systems in

6. CONCLUSION

The Wöhler curves represented by linguistic equation (LE) models are feasible in calculating the contributions of a complex load that varies with time. Torque measurements collected from a rolling mill are analysed with a combination of two norms and scaled with the nonlinear scaling approach. The stress index is linked to the fatigue by a linguistic S-N curve, which is linear. The high stress cases are seen as a very steep rise in the semilogarithmic curve. At the risk level higher than 60%, a single high torque level can have a strong effect on the activation of a failure. Long operating periods can be achieved if the risk levels are low. A generalised SPC approach is a feasible solution to demonstrate the risky stress levels during the operation. These approaches operate well for the limited set of failures analysed in this study and is promising for practical use.

ACKNOWLEDGEMENTS

This work was done within the research program Measurement, Monitoring and Environmental Efficiency Assessment (MMEA) on the basis of the project Integrated condition-based control and maintenance (ICBCOM). The funding of the TEKES (the Finnish Funding Agency for Technology and Innovation) and Outokumpu Stainless Oy is acknowledged.

REFERENCES


