

# Generalised spectral norms – a method for automatic condition monitoring

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## ABSTRACT

Condition monitoring is usually based on measurements of vibration, or some other physical phenomenon. When a change in signal is observed, it is often considered a sign of a deteriorated condition of the target. It is quite common, that some kind of features are used for monitoring, root mean square and peak values probably being the most used commonly ones. These features are actually special cases of the weighted  $\bar{l}_p$  norm, which is in principle the very same mathematical method as the spectral  $\bar{l}_p$  norm. However, the spectral norms have an advantage that they can provide information about the frequency content, which traditional weighted norms are unable to do. This paper discusses the concept of spectral norms, which we suggest to be a potential method for automatic condition monitoring in the future.

## 1 INTRODUCTION

Visually inspecting the time domain signal and the frequency spectrum are examples of ways to determine whether or not the signal shows a change. These methods are quite commonly applied in industrial condition monitoring, because they are very simple to perform with modern measurement tools.

However, visual analysis of the frequency spectrum or time domain signal are not always optimal solutions. This type of actions consume relatively great amount of manpower and require a quite experienced operator. In addition, the probability of erroneous interpretation may be significant. In order to provide easily understandable information, different features are often calculated. In industry, the most common ones are the root mean square (RMS) and peak values. Nonetheless, it has been shown in number of different studies [8, 9, 10, 12], that these features are often inadequate, especially when early fault detection is desired. Peak and RMS values are in fact special cases of the  $l_p$  norms.

Several frequency ranges need to be analysed separately in the condition monitoring of complex structures, e.g. epicyclic gearboxes which consist of one or more outer gears, or planet gears, revolving around a central, or sun gear. Rotation frequencies can be calculated for different gear components, see [3]

This paper focuses on spectral extensions of the generalised norms and demonstrates their operation in combined time and frequency domain analysis.

## 2 THE $\bar{l}_p$ NORMS

The weighted  $l_p$  norm is defined by

$$\|x^{(\alpha)}\|_{p,w} = \left( \sum_{i=1}^N w_i |x_i^{(\alpha)}|^p \right)^{\frac{1}{p}}, \quad (1)$$

where the real number  $\alpha$  is the order of derivative,  $w_i$  is a weighting factor,  $x$  is the displacement,  $N$  is the number of samples in signal, and the real number  $p > 0$  the order of norm.

The weighted  $\bar{l}_p$  norm was presented by Lahdelma in /11/. This has shown enormous potential in condition monitoring /4, 6, 7, 8, 9, 10/, and it is defined by:

$$\|\bar{x}^{(\alpha)}\|_p \equiv \left( \frac{1}{N} \sum_{i=1}^N |x_i^{(\alpha)}|^p \right)^{\frac{1}{p}} = \left( \frac{1}{N} \right)^{\frac{1}{p}} \|x_i^{(\alpha)}\|_p. \quad (2)$$

Furthermore, it is possible to calculate MIT indices /8/, which can be used to create an easily understandable way of determining the relative change in the value of the norm. The MIT index is calculated by

$$\tau_{MIT_{\alpha_1, \alpha_2, \dots, \alpha_n}^{p_1, p_2, \dots, p_n}} = \frac{1}{n} \sum_{i=1}^n b_{\alpha_i} \frac{\|\bar{x}^{(\alpha_i)}\|_{p_i}}{(\|\bar{x}^{(\alpha_i)}\|_{p_i})_0}. \quad (3)$$

The  $\bar{l}_p$  norms and MIT indices are effective calculation methods in condition monitoring, but when they are calculated using the time domain signal as input data, no information, on which frequencies the changes of the signal have occurred is produced.

This is where the generalised spectral norms can be used. The spectral  $\bar{l}_p$  norm is defined precisely as in (2), but in case of the spectral norm we replace  $x_i$  from the time domain with complex numbers representing frequency domain components, which we obtain from the original time domain signal by applying the Discrete Fourier Transform (DFT) on it. The DFT is nowadays commonly calculated using Fast Fourier Transform (FFT) algorithm, because FFT is easily available in several programming languages and software tools. It should be noted as well, that even though the MIT index was originally intended to be used on domain signals, it can be calculated in exactly the same manner in case of the spectral norms as it is done for time domain norms.

When we perform calculations described above, we create a norm which gives us information on the precise frequencies selected. This technique could potentially be used in automatic condition monitoring, because in this case operator often wants to monitor only certain frequencies, which are known to be able to indicate certain faults. For instance, it is quite common practice to calculate the frequencies, which are expected to be changed in case of different roller bearing faults. Applying the spectral kurtosis in this case would mean that we can calculate an easily understandable feature for each type of roller bearing fault, which can be used as an input of an automatic monitoring system. Thus the system could indicate which frequencies have been subject to change, in addition to traditional monitoring of overall levels. A more advanced system could even produce a message, which would determine the probable type and location of the fault.

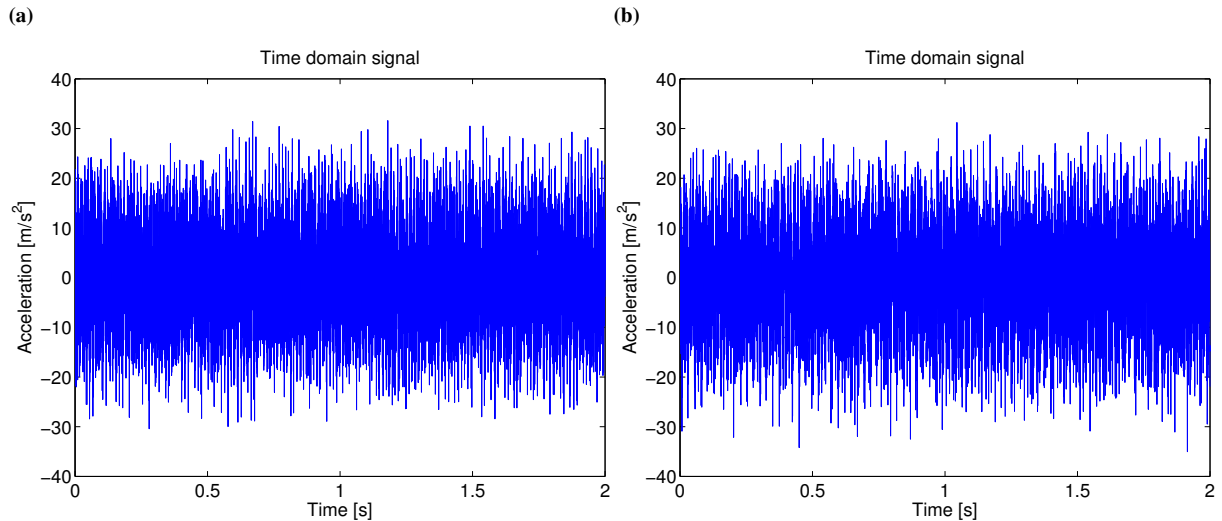
The concept of the spectral norms was first presented in /5/. Spectral kurtosis presented by Dwyer in 1983 /2/ has some similarities to the spectral  $\bar{l}_p$  norms. The spectral kurtosis has been applied to condition monitoring in some studies /1, 13/.

### 3 DEMONSTRATION OF USE OF THE SPECTRAL NORMS

In the time domain the weighted  $\bar{l}_p$  norms are quite often applied in condition monitoring. The norm  $\bar{l}_2$  is RMS value and  $\bar{l}_\infty$  is the peak value. Both of these can be found on most of the modern commercially available condition monitoring systems. Unfortunately, other orders of norms are very rarely applied nowadays, despite the fact there are several research results which indicate that these norms could quite often be feasible. /9, 10/

In following, we show how the spectral  $\bar{l}_p$  norms provide a simple piece of information about changes in specific frequency contents of the signal. To demonstrate how spectral norms could be used in condition monitoring, we have here applied this technique to some artificially generated signals. For the demonstration, we created 2 signals which include 3 sinusoidal components at 400 Hz, 800 Hz and 1200 Hz which we here consider the interesting ones. Besides we added 4 other sines at random frequencies. Amplitudes and phases of these sines were defined randomly, in order to create some differences to signals. In addition, we added noise to both of the signals, to make the signal more similar to signals one can obtain by measurements.

The time domain signals are shown in Figure 1. It is a fairly straightforward conclusion, that there is practically no difference that could be noticed between Figure 1 a) and Figure 1 b).



**Figure 1.** The two time domain signals.

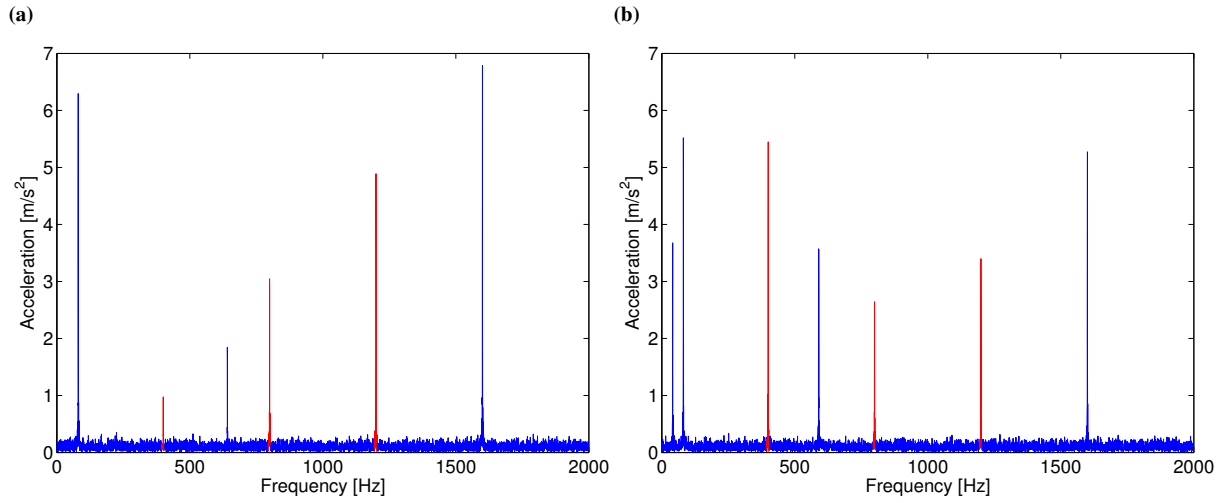
By taking a brief look at Table 1 where the time domain  $\bar{l}_p$  norms are presented, it is easy to conclude that the change is negligible. When it is question about a signal which is created by calculations, we can consider a difference around 3% as a minor difference, but in case of actual condition monitoring, or perhaps even the most measured signals in general, the difference around this could hardly be considered a difference at all.

**Table 1.** Time domain norms.

	Signal a)	Signal b)	Relative change
$\ \bar{x}^{(2)}\ _2$	$9.896 m/s^2$	$10.184 m/s^2$	1.03
$\ \bar{x}^{(2)}\ _4$	$12.769 m/s^2$	$13.128 m/s^2$	1.03
$\ \bar{x}^{(2)}\ _8$	$16.657 m/s^2$	$17.142 m/s^2$	1.03

In Figure 2 are the frequency spectra from the time domain signals shown in Figure 1. The frequencies which are included in the spectral norms shown later have been highlighted with red colour.

At least when the certain frequencies are highlighted, it is quite easy to state that there are some changes at these frequencies. However, it is quite difficult to say which of the frequencies has changed, and how great the change actually is just by visually analysing the spectrum.



**Figure 2.** The frequency spectra of the signals. The frequencies which have been included in the spectral norms are coloured red.

The spectral norms shown in Tables 2 and 3 are calculated from the 10 Hz wide frequency ranges around the frequencies 400 Hz, 800 Hz and 1200 Hz. In Table 2 the norms are calculated separately for each frequency range, and in table 3 the norms are calculated by processing the three sequences of the frequency spectrum as a concatenated continuous sequence. This is to say that in the case of Table 2 the norms give information about each frequency range separately, and in Table 3 the norms indicate the change of all of three frequency ranges in overall.

**Table 2.** The spectral norms.

	Signal a)	Signal b)	Relative change
$\ \bar{x}^{(2)}\ _2 @ 400Hz$	$0.243m/s^2$	$1.232m/s^2$	5.07
$\ \bar{x}^{(2)}\ _4 @ 400Hz$	$0.432m/s^2$	$2.402m/s^2$	5.56
$\ \bar{x}^{(2)}\ _8 @ 400Hz$	$0.633m/s^2$	$3.535m/s^2$	5.58
$\ \bar{x}^{(2)}\ _2 @ 800Hz$	$0.753m/s^2$	$0.669m/s^2$	0.88
$\ \bar{x}^{(2)}\ _4 @ 800Hz$	$1.437m/s^2$	$1.257m/s^2$	0.87
$\ \bar{x}^{(2)}\ _8 @ 800Hz$	$2.054m/s^2$	$1.794m/s^2$	0.87
$\ \bar{x}^{(2)}\ _2 @ 1200Hz$	$1.079m/s^2$	$0.757m/s^2$	0.70
$\ \bar{x}^{(2)}\ _4 @ 1200Hz$	$2.100m/s^2$	$1.467m/s^2$	0.70
$\ \bar{x}^{(2)}\ _8 @ 1200Hz$	$3.147m/s^2$	$2.190m/s^2$	0.70

It should be noted that the values of relative change in Table 2 are roughly the same as the changes of height of the highest red peaks when comparing Figure 2 a) with Figure 2 b). This is because the norm is calculated from a part of spectrum, where a single peak is significantly higher than the rest of the spectral lines, and thus this peak has a dominant effect in the value of the norm.

**Table 3.** The combined spectral norms.

	Signal a)	Signal b)	Relative change
$\ \bar{x}^{(2)}\ _2$	$0.773m/s^2$	$0.922m/s^2$	1.19
$\ \bar{x}^{(2)}\ _4$	$1.679m/s^2$	$1.920m/s^2$	1.14
$\ \bar{x}^{(2)}\ _8$	$2.757m/s^2$	$3.095m/s^2$	1.12

The key point about the information in Tables 2 and 3 is that the spectral  $\bar{l}_p$  norms are able to produce an easily interpretable indication if the certain frequency content of the signal has changed.

## 4 SUGGESTED APPLICATIONS

We see several potential applications for the spectral  $\bar{l}_p$  norms. In condition monitoring, a lot of different faults are often considered causing changes in certain frequency. As mentioned before, these changes may often be possible to detect by simply visually inspecting the spectrum. Using some alarm limits which any of the frequency components in certain frequency range may not exceed is fairly common as well.

Visually evaluating the changes is obviously more time consuming, and has a greater risk of human error, than using a norm which can be calculated automatically. Furthermore, it is a lot easier to see how great the change in certain frequency range has been by reading out a simple numeric value, than looking at the frequency spectrum. It is possible as well to determine warning and alarm limits to the values of the norms, thus creating “traffic light” system which is very easy to use even for an inexperienced operator.

For the reasons stated above, we suggest that the spectral  $\bar{l}_p$  norms could be applied in automatic, or semi-automatic, condition monitoring, when looking for signs for such faults as misalignment, unbalance or defects in roller bearings. In some cases it might be more useful to calculate the spectral norms for an envelope spectrum, than the amplitude spectrum. In this case, the norms are useful to evaluate the change in some very precisely defined frequencies, but similar information about an overall change in a wider frequency range is possible to achieve. This might be useful for instance when trying to detect resonance in structures. Applying the spectral norms to the spectrum of higher order derivatives than acceleration could be interesting as well. This will probably be one topic of our future research.

## 5 CONCLUSIONS

Generalised spectral norms include information about the frequency content within the time domain analysis and thus improve possibilities to detect faults which have earlier required time consuming studies of the frequency spectra.

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